7

Patterns and algebra

A seed you have planted in a pot is starting to grow. When you first notice the seedling, it measures 2.4 centimetres high. One day later the seedling is 3.8 centimetres high, the following day it is 5.2 centimetres and on the third day it is 6.6 centimetres high. If the seedling continues to grow at the same rate, how tall will it be after 7 days? How tall will the plant be after 18 days?

In this chapter, you will be using rules or formulas involving pronumerals to represent numbers to help solve problems like this more easily.

Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest 7* CD-ROM or ask your teacher for a copy.

SkillSHEET 7.1

1 For each of the following sequences of numbers, describe the pattern in words and then write down the next three numbers in the pattern.

a 7, 9, 11, 13, …
b 28, 24, 20, 16, …
c 3, 6, 12, 24, …
d 100 000, 10 000, 1000, …

SkillSHEET 7.2

2 Use 8 as your starting number in each case and find the result when you follow each instruction.

a Multiply by 2
b Add 15
c Subtract 6
d Divide by 4

SkillSHEET 7.3

3 Describe each number pattern shown in the tables.

a

First number	1	2	3	4	5
Second number	7	8	9	10	11

b

First number	1	3	5	7	9
Second number	3	9	15	21	27

SkillSHEET 7.5

4 **a** Complete the table shown, using the diagrams below as a guide.

1 square   2 squares   3 squares

Number of squares	1	2	3	4	5	6
Number of sides	4	8				

b How many sides are there for 10 squares?

SkillSHEET 7.6

5 Rewrite each of the following as a mathematical sentence (that is, rewrite each using numbers and one of the four operations +, −, × or ÷).

a The sum of 3 and 5
b The product of 7 and 8
c The difference between 6 and 2
d 9 more than 5

Using rules

We often use mathematics to describe relationships in the world about us. Mathematicians study these patterns found in nature to discover rules that describe how the relationships are produced. These rules can then be applied in other, more general situations.

Patterns and rules

Let's look at some patterns in numbers and shapes, and see if we can discover the rules that describe them.

Number patterns

If we look at the number pattern 1, 4, 7, 10, … we can see that by adding 3 to any of these numbers, we obtain the next number. Hence, the next three numbers would be 13, 16 and 19. This number pattern is called a *sequence*. Each number in the sequence is called a *term*. This sequence (like all sequences) has a *rule* that describes the pattern. The rule here is 'add 3'.

SkillSHEET 7.1

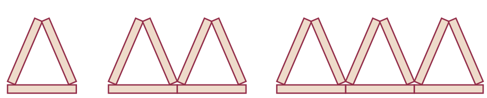
Number patterns

Copy the patterns below, describe the pattern in words and then write down the next three numbers in the pattern.

1. 2, 4, 6, 8, …
2. 3, 8, 13, 18, …
3. 27, 24, 21, 18, …
4. 1, 3, 9, 27, …
5. 128, 64, 32, 16, …
6. 1, 4, 9, 16, …

Geometric patterns

We can also find patterns in geometric shapes. If we examine the three shapes below, we can see patterns by investigating the changes from one shape to the next — for example, look at the number of matchsticks in each set of triangles.



By using a table of values we can see a number pattern developing:

Number of triangles	1	2	3	4	5	6
Number of matchsticks	3	6	9	12	15	18

The pattern in the bottom row is 3, 6, 9, . . .; we can see that the rule here is 'add 3'.

We can also look for a relationship between the number of triangles and the number of matchsticks in each shape. If you examine the table, you will see that a relationship can be found. In words, the relationship is 'the number of matchsticks equals 3 times the number of triangles'.

Geometric patterns

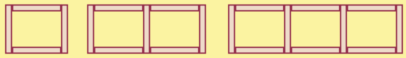
For each of the sets of shapes below:

a Construct a table, as shown on the previous page, to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.

b Devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.

c Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.

1



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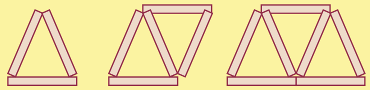

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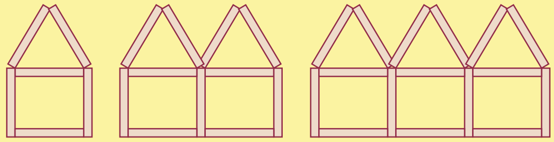

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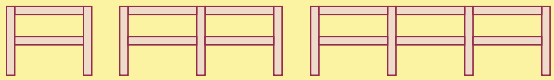
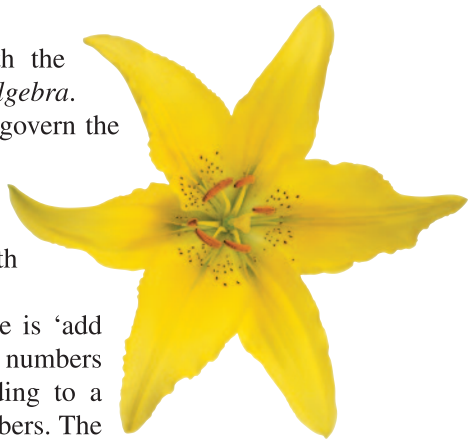


Algebra

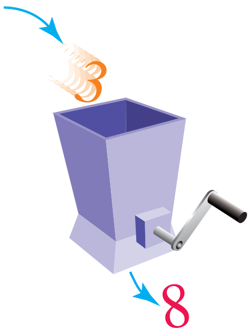


The branch of mathematics that deals with the patterns and rules discussed above is called *algebra*.

It allows us to investigate the rules which govern the behaviour of everyday things, such as the arrangement of petals in flowers, the speed that a ball will reach when it is dropped from a certain height and the movement of the earth around the sun.

An example of a simple mathematical rule is 'add 5'. Imagine an 'algebra machine' which takes numbers (input numbers) and processes them according to a particular rule to produce a set of output numbers. The input and output numbers resulting from this rule are shown below.

Input numbers	Rule	Output numbers
3, 5, 89, 222	Add 5	8, 10, 94, 227



The algebra machine takes any input number and changes it according to the rule given. In this case, 5 is added to each input number. Any number can be used as an input number. The algebra machine might print out a table like this:

Input	3	5	89	222
Output	8	10	94	227

The machine can also be reprogrammed to work with different rules. The rule can be as simple or as complicated as you like. For example, you could change the rule to 'multiply each input number by 2' or 'subtract 26 from each input number, then multiply by 143'.

In algebra a rule can be used to produce a set of output numbers from a set of input numbers.

WORKED Example 1

Complete the table at right using the given rule to work out the correct output numbers.
Rule: Subtract 3 from each input number.

Input	4	10	37	143
Output				

THINK

1. Take the first input number (4) and apply the rule. The rule tells you to subtract 3.
2. Repeat for the other input numbers.
3. Enter these output values in the table.

WRITE

$4 - 3 = 1$

$10 - 3 = 7$
$37 - 3 = 34$
$143 - 3 = 140$

Input	4	10	37	143
Output	1	7	34	140

WORKED Example 2

Complete the table below by using the following rule to work out the correct output numbers in each case.
Rule: Multiply each input number by 8, then subtract 2.

Input	3	1	5	24
Output				

THINK

1. Consider the first input number (3) and apply the rule.
2. The first part of the rule tells you to multiply by 8.

WRITE

$3 \times 8 = 24$

Continued over page

THINK

3. The second part of the rule says subtract 2 from your answer. Enter this value (22) into the table.
4. Repeat for the other input numbers.
5. Enter the output values in the table.

WRITE

$24 - 2 = 22$

$1 \times 8 = 8 \qquad 8 - 2 = 6$
$5 \times 8 = 40 \qquad 40 - 2 = 38$
$24 \times 8 = 192 \qquad 192 - 2 = 190$

Input	3	1	5	24
Output	22	6	38	190

WORKED Example 3

Use the rule given below to work out the missing input and output numbers.
Rule: Add 4 to each input number.

Input	1	5	25			
Output				6	38	190

THINK

1. Consider the input numbers that are given. The rule tells you to add 4.
2. Consider the output numbers that are given. The rule has been used to add 4, so do the opposite and subtract 4.
3. Enter the new output and input values in the table.

WRITE

$1 + 4 = 5$
$5 + 4 = 9$
$25 + 4 = 29$
$6 - 4 = 2$
$38 - 4 = 34$
$190 - 4 = 186$

Input	1	5	25	2	34	186
Output	5	9	29	6	38	190

remember

1. By observing number patterns, we can discover the rules by which a pattern is produced.
2. In algebra, a rule can be used to produce a set of output numbers from a set of input numbers.

EXERCISE 7A Using rules

WORKED Example 1

SkillSHEET 7.2

1 Copy and complete the following tables. Use the rule given in each case to work out the correct output numbers.

a Subtract 4 from each input number.

Input	10	5	6	14	4
Output					

b Add 1 to each input number.

Input	1	13	6	107	2
Output					

c Add 12 to each input number.

Input	10	51	60	1	144
Output					

d Divide each input number by 3.

Input	12	3	66	21	141
Output					

WORKED Example 2

2 Copy and complete the following tables. Use the rule given in each case to work out the correct output numbers.

a Multiply each input number by 2, then add 5.

Input	3	4	2	10	17
Output					

b Multiply each input number by 8, then subtract 4.

Input	4	2	5	20	100
Output					

c Add 3 to each input number, then multiply by 5.

Input	5	2	12	3	43
Output					

d Subtract 3 from each input number, then multiply by 11.

Input	5	8	25	3	4
Output					

e Multiply each input number by itself.

Input	5	8	10	0	1
Output					

f Multiply each input number by itself, then add 4.

Input	4	7	12	0	1
Output					

WORKED Example 3

3 Copy and complete the following tables. Use the rule given in each case to work out the missing input and output numbers.

a Add 2 to each input number.

Input	1	3	27			
Output				4	55	193

b Add 20 to each input number.

Input	3	56	25			
Output				94	20	1773

c Subtract 5 from each input number.

Input	7	96	15			
Output				4	12	104

d Multiply each input number by 4.

Input	1	6	321			
Output				8	28	412

e Multiply each input number by 2, then add 2.

Input	3	13	21			
Output				22	4	102

f Multiply each input number by 5, then subtract 7.

Input	2	16	5			
Output				8	18	93

g Divide each input number by 3.

Input	3	15	273			
Output				2	54	21

h Multiply each input number by itself.

Input	5	17	1			
Output				49	144	4

i Multiply each input number by 20, then subtract 6.

Input	2	15	7			
Output				54	214	174

j Subtract 3 from each input number, then divide by 4.

Input	19	31	7			
Output				12	15	5

MATHS QUEST CHALLENGE

1 Using the six consecutive numbers from 4 to 9, complete the magic square at right so that each row, column and diagonal totals 15.

	1	
3		
		2

2 Using 20 cubes, make four piles so that the first pile contains 4 more cubes than the second pile, the second pile contains 1 cube less than the third pile, and the fourth pile contains twice as many cubes as the second pile. How many cubes are in each pile?

History of mathematics

THE DEVELOPMENT OF ALGEBRA

Algebra has been used by mathematicians for thousands of years. The earliest recorded use of algebra is found in clay tablets from the Mesopotamian civilisation which are over 3500 years old. The Mesopotamians, who lived in the fertile valley between the Tigris and Euphrates rivers, in what is now Iraq, used algebra to solve many practical problems.

In the seventh century AD the Eastern Roman Empire, whose capital was Byzantium (now Istanbul), was conquered by the Arabs, who were followers of the new prophet Mohammed (AD 570–632). The Muslims had a strong trading tradition and they were interested in mathematics as a way of calculating shares in goods and transactions. In AD 830, Al-Khwarizmi, one of the greatest Arab mathematicians, published a book called *Al-jabr wa'l muqabala.* Al-jabr, meaning 'reunion of broken parts' or 'balance', is the word from which the modern term 'algebra' is derived.

One of Al-Khwarizmi's famous successors was the Persian poet and mathematician Omar Khayyam (shown below left), who was also interested in equations. Knowledge of algebra came to Europe from Latin translations of Al-Khwarizmi's work.

Centuries later, the English scientist and mathematician Isaac Newton (1642–1727) showed that the behaviour of most things in the known universe could be described by mathematical rules involving algebra. It is only in the last 400 years that we have developed symbols to use with algebraic problems. Before then, words and sentences were used.

Questions

1. What is the origin of the word 'algebra'?
2. What is the name of the Persian poet who followed after Al-Khwarizmi?
3. What is the modern name for the city of Byzantium?
4. Which country now occupies the area formerly known as Mesopotamia?

Writing and finding formulas

One of the features of algebra which distinguishes it from other branches of mathematics is the use of letters or *pronumerals*.

A pronumeral is a letter or symbol that is used in place of a number.

Pronumerals allow us to write a rule more simply. Look at the following table.

Input	1	2	3	4	20
Output	4	5	6	7	23

The rule for this table can be written as output = input + 3. If we use the letter a to represent the input numbers and the letter b to represent the output numbers, the table looks like the one on the right.

a	1	2	3	4	20
b	4	5	6	7	23

The rule can now be written more simply as $b = a + 3$. When the rule is written using pronumerals like this, it is called a *formula*.

Many students find algebra confusing because of the way letters are chosen at random to represent numbers. Any letter from the alphabet can be used to represent the input and output numbers for the algebra machine. For example, we could use x for the input numbers and m for the output numbers, making the formula $m = x + 3$. The actual letter you choose is not important; what is important is that the rule is shown correctly. When you write a formula using pronumerals, be careful to put the input and output numbers in the right place. For example, $x = m + 3$ is not the same formula as $m = x + 3$.

The value of a pronumeral is not determined by its place in the alphabet. If we use b to represent a number, this does not necessarily mean that it is a smaller number than x or c. When multiplying numbers and pronumerals, we don't need to show the multiplication sign. We place numbers in front of pronumerals, so $h = k \times 5$ is written as $h = 5k$.

WORKED Example 4

Rewrite the following formulas, leaving out the multiplication (×) sign.

a $m = q \times 4 + 3$ **b** $b = (m + 2) \times 5$ **c** $g = (2 \times w - 6) \times 3$

THINK	WRITE
a 1 Write down the formula.	**a** $m = q \times 4 + 3$
2 Leave out the multiplication sign. (Remember to write the number being multiplied with the pronumeral in front of the pronumeral.)	$m = 4q + 3$
b 1 Write down the formula.	**b** $b = (m + 2) \times 5$
2 Leave out the multiplication sign. (Remember to write the number being multiplied in front of the pronumeral.)	$b = 5(m + 2)$
c 1 Write down the formula.	**c** $g = (2 \times w - 6) \times 3$
2 Leave out the multiplication signs. (Remember to write the number being multiplied in front of the pronumeral.)	$g = 3(2w - 6)$

WORKED Example 5

Look at the table at right and complete the formula by inserting a number in the gap.
$g = f + \_\_$

f	3	6	8	13	20
g	5	8	10	15	22

THINK	WRITE
1 Look at the first pair of numbers, 3 and 5. Look at the formula and try to guess the number which must be added to 3 to get 5.	$5 = 3 + 2$
2 Look at the next pair of numbers to see if adding 2 works again.	$8 = 6 + 2$
3 Check the other number pairs to see if this works every time.	$10 = 8 + 2$ $15 = 13 + 2$ $22 = 20 + 2$
4 Write down the rule.	$g = f + 2$

WORKED Example 6

Use the pronumerals given to write a formula for the table at right.

a	6	5	4	13	7
b	7	5	3	21	9

THINK	WRITE
1 Look at the first pair of numbers, 6 and 7. Use a system of 'guess and check' to come up with operations (add, subtract, multiply or divide) which will give the right result. Your first guess might be 'add one to *a*'.	$a = 6,\ b = 7 \quad 6 + 1 = 7$
2 Check this guess on the next pair of numbers, 5 and 5. It does not work!	$a = 5,\ b = 5 \quad 5 + 1$ does not equal 5.
3 Try another operation or combination of operations, such as multiply by 2, then subtract 5. (You may need to try several different operations before finding the right one.)	$a = 6,\ b = 7 \quad 2 \times 6 = 12,\ 12 - 5 = 7$
4 Again, check the other number pairs. This rule or formula works for all the pairs of numbers.	$a = 5,\ b = 5 \quad 2 \times 5 = 10,\ 10 - 5 = 5$ $a = 4,\ b = 3 \quad 2 \times 4 = 8,\ 8 - 5 = 3$ $a = 13,\ b = 21 \quad 2 \times 13 = 26,\ 26 - 5 = 21$ $a = 7,\ b = 9 \quad 2 \times 7 = 14,\ 14 - 5 = 9$
5 Write down the rule carefully, being sure to put the input and output numbers in the right places.	$b = 2a - 5$

remember

1. A *pronumeral* is a letter or symbol that is used in place of a number.
2. When a rule is written using pronumerals it is called a *formula*.
3. When multiplying numbers and pronumerals we don't need to show the multiplication sign, and we place numbers being multiplied in front of pronumerals.

EXERCISE 7B Writing and finding formulas

WORKED Example 4

1 Rewrite the following formulas, leaving out the multiplication (×) sign.

a $b = 4 \times h$ **b** $m = f \times 4$ **c** $r = a \times 5$
d $m = t \times 4$ **e** $x = (k + 4) \times 5$ **f** $k = 6 \times w - 2$
g $t = 4 \times (20 - g)$ **h** $b = 10 \times a - 5$ **i** $d = 6 \times f + 7$
j $h = (x + 5) \times 9$ **k** $y = (b \times 3 + 6) \times 8$ **l** $y = 8 \times p - 6$
m $g = 2 \times (3 \times r + 17)$ **n** $j = (h \times 5 - 4) \times 18$

SkillSHEET 7.3

WORKED Example 5

2 Look at the following tables and complete the formula for each table by inserting a number in the gap.

a

f	3	1	6	8	0
g	12	10	15	17	9

$g = f +$ ____

b

a	7	8	3	4	11
b	4	5	0	1	8

$b = a -$ ____

c

k	5	6	12	1	3
t	11	13	25	3	7

$t = 2k +$ ____

d

x	5	8	1	3	11
y	30	51	2	16	72

$y = 7x -$ ____

e

a	1	9	4	12	6
g	6	38	18	50	26

$g = 4a +$ ____

f

m	3	1	11	2	4
t	38	20	110	29	47

$t = 9m +$ ____

g

p	4	13	5	0	6
w	14	41	17	2	20

$w =$ ____ $\times p + 2$

h

t	2	3	9	12	7
x	2	7	37	52	27

$x =$ ____ $\times t - 8$

i

m	4	1	15	7	2
p	11	2	44	20	5

$p = 3m -$ ____

j

s	1	6	12	5	7
b	3	53	113	43	63

$b = s \times$ ____ $- 7$

WORKED Example 6

3 Look at the following tables and use the pronumerals given to write a formula for each table.

GC program—Casio: Finding the formula

GC program—TI: Finding the formula

a

a	4	2	5	12	8
b	7	5	8	15	11

b

t	1	8	3	2	15
w	6	13	8	7	20

c

m	5	12	11	7	4
a	1	8	7	3	0

d

s	2	12	5	0	1
t	7	27	13	3	5

e

f	3	1	11	4	6
g	5	1	21	7	11

f

s	1	9	3	12	7
c	4	28	10	37	22

g

d	7	4	2	5	12
a	25	13	5	17	45

h

s	1	6	2	5	10
g	1	26	6	21	46

i

f	3	2	8	11	4
e	13	2	68	101	24

j

p	5	2	6	12	1
q	104	44	124	244	24

k

b	75	60	20	55	100
t	300	240	80	220	400

l

u	10	20	100	7 000	5
p	29	59	299	20 999	14

For questions **4** to **12**, a formula has been used by an algebra machine to produce each table.

4 **multiple choice**

a	3	6	36	0	1
b	5	8	38	2	3

The formula used is:

A $b = a + 5$ **B** $b = 3a$ **C** $b = a + 2$ **D** $b = a - 2$

5 **multiple choice**

g	1	2	18	56	4
k	0	1	17	55	3

The formula used is:

A $k = g + 1$ **B** $k = 2g$ **C** $k = g - 1$ **D** $k = 2g - 2$

6 **multiple choice**

r	5	8	17	6	9
w	0	3	12	1	4

The formula used is:

A $w = r + 5$ **B** $w = 5r$ **C** $w = 2r - 5$ **D** $w = r - 5$

7 **multiple choice**

p	0	6	21	7	9
m	0	12	42	14	18

The formula used is:

A $m = p$ **B** $m = p + 6$ **C** $m = p + 2$ **D** $m = 2p$

8 **multiple choice**

m	1	2	7	98	5
w	8	9	14	105	12

The formula used is:

A $w = 8m$ **B** $w = 2m + 6$ **C** $w = m + 7$ **D** $w = 2m + 5$

9 **multiple choice**

p	2	11	78	4	3
t	8	35	236	14	11

The formula used is:

A $t = p + 6$ **B** $t = 2p + 3$ **C** $t = 4p$ **D** $t = 3p + 2$

10 **multiple choice**

g	2	11	78	4	3
k	10	55	390	20	15

The formula used is:

A $k = 5g$ **B** $k = 4g + 2$ **C** $k = g + 8$ **D** $k = 2g + 6$

11 **multiple choice**

b	1	2	65	9	14
a	9	11	137	25	35

The formula used is:

A $a = 8b + 1$ **B** $a = 3b + 5$ **C** $a = b + 8$ **D** $a = 2b + 7$

12 **multiple choice**

m	7	5	3	2	11
e	85	61	37	25	133

The formula used is:

A $e = m + 78$ **B** $e = 10m + 11$ **C** $e = 12m + 1$ **D** $e = 6m + 43$

WorkSHEET 7.1

10 QUICK QUESTIONS 1

1 Complete the following table using the rule given. Rule: Add 4 to each input number.

Input	2	5	14	20	23	36
Output						

2 Complete the following table using the rule given. Rule: Subtract 2 from each input number, then multiply by 3.

Input	6	24	11	16	32	8
Output						

3 Complete the following table using the rule given to work out the missing input and output numbers. Rule: Divide each input number by 5.

Input	65	30	40			
Output				5	14	3

4 Find the formula used by the algebra machine to complete this table.

Input	1	2	3	4	5	6
Output	7	8	9	10	11	12

5 Find the formula used by the algebra machine and then use this to complete the rest of the table.

Input	4	13	7	12	1	18	24	31
Output	16	52	28	48				

6 Find the formula used by the algebra machine to complete this table. (*Hint*: The rule has two steps.)

Input	2	10	8	16	23	35
Output	15	55	45	85	120	180

7 Rewrite $k = 8 \times w + 12$ without a multiplication sign.

8 Rewrite $g = 7 \times m - 4 \times (t + 11)$ without multiplication signs.

9 Complete the rule for the table by inserting a number in the space provided.

a	5	8	11	4	26	31
b	3	6	9	2	24	29

$b = a - \_\_\_$

10 Complete the rule for the table by inserting a number in the space provided.

t	4	11	25	17	6	10
m	20	48	104	72	28	44

$m = 4t + \_\_\_$

Substitution

If we know the rule or formula for a particular table, we can take on the role of the 'algebra machine' and find the numbers that are needed to fill any gaps in the table. Look at the table below.

g	7	5	3	2	11
e	33	23	13		

Were you able to work out that the formula (rule) for this table is $e = 5g - 2$? This formula can be used to find the missing numbers in the table.

For example, to find the first missing 'e' number, write the formula: $e = 5g - 2$. Now replace g with its value from the table; that is, 2. This gives:

$$\begin{aligned} e &= 5 \times 2 - 2 \text{ (remember, } 5g \text{ means } 5 \times g) \\ &= 10 - 2 \\ &= 8 \end{aligned}$$

The other gap in the table can be filled the same way.

$$\begin{aligned} e &= 5g - 2 \\ e &= 5 \times 11 - 2 \\ &= 55 - 2 \\ &= 53 \end{aligned}$$

If the value of g is 20, let us consider how we can find the value of e. First we would write down the rule to find e and then we replace g with the given value of 20.

$$\begin{aligned} e &= 5g - 2 \\ \text{If } g &= 20 \\ e &= 5 \times 20 - 2 \\ &= 100 - 2 \\ &= 98 \end{aligned}$$

When a pronumeral in a formula is replaced by a number, we say that the number is *substituted* into the formula.

In the example above, g was replaced by 20; that is, 20 was substituted for g. Numbers can be substituted for pronumerals whenever the formula is known. Look at the example below:

WORKED Example 7

If $x = 3t - 6$, substitute the given values of t into the formula to find the value of x in each case.

a $t = 5$ **b** $t = 12$

THINK	WRITE
a 1 Write the formula.	**a** $x = 3t - 6$
2 Substitute 5 for t.	If $t = 5$
	$x = 3 \times 5 - 6$
3 Work out 3×5.	$x = 15 - 6$
4 Subtract 6.	$x = 9$

THINK

b 1 Write the formula again.
2 Substitute 12 for t.
3 Work out 3×12.
4 Subtract 6.

WRITE

b $x = 3t - 6$
If $t = 12$
$x = 3 \times 12 - 6$
$x = 36 - 6$
$x = 30$

WORKED Example 8

Find the value of m by substituting the given value of x into the formula below.
$m = 3(2x + 3)$, $x = 4$

THINK

1 Write the formula.
2 Substitute 4 for x.
3 Work inside the brackets first, remembering that $2x$ means $2 \times x$.
4 Multiply by the number outside the brackets.

WRITE

$m = 3(2x + 3)$
If $x = 4$
$m = 3(2 \times 4 + 3)$
$m = 3(11)$
$m = 3 \times 11$
$= 33$

remember

When a pronumeral in a formula is replaced by a number, we say that the number is substituted into the formula.

EXERCISE 7C Substitution

1 Use the given rule to complete the table in each case.

a $y = x + 5$

x	3	2	18	11	40
y					

b $m = n - 17$

n	17	43	20	700	22
m					

c $y = x + 1$

x	3	123	70	40	1
y					

d $p = 2h + 4$

h	5	22	62	1	11
p					

e $t = 3b - 7$

b	5	60	20	15	100
t					

f $s = 4t + 4$

t	1	8	40	7	3
s					

g $y = x - 1$

x	3	123	70	40	1
y					

h $s = 10g - 8$

g	19	8	7	154	1
s					

i $t = 12 - m$

m	3	11	7	4	12
t					

j $p = 6f$

f	1	111	11	12	8
p					

k $m = 12 - 2y$

y	3	1	6	4	0
m					

l $k = 12r - 30$

r	4	11	12	16	804
k					

WORKED Example 7

2 Substitute the given values into each formula to find the value of m in each case.

a $m = g - 2$
- **i** $g = 4$ **ii** $g = 5$ **iii** $g = 2$ **iv** $g = 102$

b $m = 2t - 3$
- **i** $t = 7$ **ii** $t = 2$ **iii** $t = 100$ **iv** $t = 8$

c $m = 12h + 7$
- **i** $h = 1$ **ii** $h = 0$ **iii** $h = 5$ **iv** $h = 20$

d $m = 10 + y$
- **i** $y = 5$ **ii** $y = 0$ **iii** $y = 2$ **iv** $y = 526$

e $m = 25 - 4w$
- **i** $w = 1$ **ii** $w = 3$ **iii** $w = 6$ **iv** $w = 0$

WORKED Example 8

3 Find the value of m by substituting the given value of the pronumeral into the formula.

a $m = 2(g + 1)$
- **i** $g = 1$ **ii** $g = 0$ **iii** $g = 12$ **iv** $g = 75$

b $m = 5(x - 2)$
- **i** $x = 6$ **ii** $x = 10$ **iii** $x = 11$ **iv** $x = 2$

c $m = 4(12 - p)$
- **i** $p = 2$ **ii** $p = 3$ **iii** $p = 12$ **iv** $p = 11$

d $m = 5t + 4t$
- **i** $t = 3$ **ii** $t = 1$ **iii** $t = 20$ **iv** $t = 0$

e $m = 5(2g - 3)$
- **i** $g = 2$ **ii** $g = 14$ **iii** $g = 5$ **iv** $g = 9$

f $m = 2(d + 2) - 3$
- **i** $d = 3$ **ii** $d = 0$ **iii** $d = 7$ **iv** $d = 31$

g $m = 3(f - 1) + 17$
- **i** $f = 1$ **ii** $f = 3$ **iii** $f = 6$ **iv** $f = 21$

h $m = 4s - s$
- **i** $s = 3$ **ii** $s = 1$ **iii** $s = 101$ **iv** $s = 72$

i $m = 3(y + 5) - 2$
  i $y = 1$ **ii** $y = 5$ **iii** $y = 0$ **iv** $y = 12$

j $m = 12(5 - e)$
  i $e = 1$ **ii** $e = 0$ **iii** $e = 5$ **iv** $e = 4$

k $m = 50 - 6v$
  i $v = 4$ **ii** $v = 7$ **iii** $v = 1$ **iv** $v = 8$

Nutrition panels

Most tinned and packaged foods that we buy have labels which list the nutritional content of the food. Most of the energy (measured in calories or kilojoules) in food comes from fat, protein and carbohydrates. It is estimated that there are 9 calories in each gram of fat and 4 calories in each gram of protein and each gram of carbohydrate.

1 From the label shown, write down the number of grams of fat, protein and carbohydrate in 100 grams of baked beans.

2 Estimate how many calories there would be in 100 grams of baked beans using the information supplied above.

3 Compare this to the value stated on the tin.

If we wanted to look at a wide variety of foods, it would be more convenient to have a rule or formula into which we could substitute numbers to easily obtain our answer.

4 In words, write a rule that could be used to calculate the number of calories in food.

5 Choose a pronumeral for each of the following quantities:
  a number of grams of fat
  b number of grams of protein
  c number of grams of carbohydrate
  d number of calories.

6 Using the pronumerals you have chosen, write a formula to calculate the number of calories.

7 The table below contains data on the fat, protein and carbohydrate content of selected foods. Copy this table into your workbook and write in your chosen pronumerals in the first row within the brackets. Use your formula to work out the calorie content of each of these foods. (The last column will be completed later on.)

Food	Number of grams of fat ( )	Number of grams of protein ( )	Number of grams of carbohydrate ( )	Number of calories ( )	Number of kilojoules ( )
1 hot cross bun	3	3	19		
100 grams of chocolate cake	16	4	56		
100 grams of roast chicken	14	26	0		
70 grams of bacon	8	21	0		
2 grilled sausages	17	13	15		
1 piece of fish (flake), no batter	1	21	0		
10 grams of sultanas	0	0	6		
25 grams of dried apricots	0	0	17		
1 banana	0	1	20		
1 apple	0	0	17		
1 serve of carrots	0	0	5		
1 serve of potatoes	0	0	17		
250 millilitres of milk	10	8	12		
210 grams of tinned tomato noodle soup	0.8	2.6	14.9		

8 Which quantity of food in this list has the highest number of calories? Is this what you expected?

9 What types of food in this list generally have a lower number of calories?

A calorie is an energy measurement unit in the imperial system. Nowadays we mostly use the metric system unit of kilojoules to measure energy. It is estimated that the number of kilojoules is equivalent to the number of calories multiplied by 4.2.

10 Use a pronumeral to represent each of the quantities 'number of calories' and 'number of kilojoules' and write a formula connecting them.

11 Complete the final column of your table by using the formula to find the number of kilojoules for each listed food. (Round your answers to the nearest kilojoule.)

12 Obtain three labels from three types of packaged food. Use the formula for converting calories to kilojoules to obtain estimates of the energy for each food item in both calories and kilojoules. Compare your answers to the data supplied on each of the three nutrition labels.

The polar bear as an athlete!

The letter beside each number pattern and the missing number give the puzzle answer code.

TOP SPEED:

15	31	125	62	27	25	51	45	58	95	43	8	648	38	37
71	44	23	30	48	16	0								

LONG JUMP:

13	55	83	76	5	53	80	160	73	22	14	39	46	9	140
11	47	57	96	2	128									

HIGH JUMP:

32	42	28	60	75	56	67	64	120

- R 7, 11, 15, 19, ___,...
- Y 19, 21, 23, 25, ___,...
- E 7, 15, 23, 31, ___,...
- R 101, 95, 89, ___,...
- E 16, 32, 48, ___,...
- P 77, 75, 73, ___,...
- O 60, 61, 59, 62, ___,...
- N 35, 70, 105, ___,...
- E 68, 70, 72, 74, ___,...
- M 150, 120, 90, ___,...
- H 51, 44, 37, ___,...
- R 1, 5, 25, ___,...
- O 8, 13, 18, 23, ___,...
- T 18, 19, 20, 21, ___,...
- H 22, 33, 44, ___,...
- E 17, 14, 11, 8, 5, ___,...
- T 20, 17, 14, 11, ___,...

- E 15, 22, 29, 36, ___,...
- O 20, 40, 60, ___,...
- T 1, 3, 5, 7, 9, 11, ___,...
- W 10, 18, 26, 34, ___,...
- F 6, 9, 12, ___, 18,...
- T 8, 20, 32, 44, ___,...
- E 41, 29, 17, ___,...
- R 6, 12, 24, 48, ___,...
- I 78, 69, 60, ___,...
- N 79, 77, 75, ___,...
- S 1, 2, 6, 24, ___,...
- O 3, 6, 12, 24, ___,...
- E 18, 20, 24, 30, ___,...
- R 79, 76, 73, 70, ___,...
- V 50, 52, 48, 54, ___,...
- K 1, 4, 9, 16, ___,...
- E 8, 21, 34, ___,...

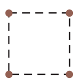
- U 128, 64, 32, ___,...
- T 80, 74, 68, ___,...
- P 3, 8, 18, 33, ___,...
- S 2, 8, 32, ___,...
- M 87, 89, 91, 93, ___,...
- T 78, 71, 64, ___,...
- E 50, 48, 46, ___,...
- R 8, 24, 72, 216, ___,...
- E 90, 89, 87, 84, 80, ___,...
- I 5, 10, 20, 40, 80, ___,...
- O 21, 22, 24, 27, ___, 36,...
- M 23, 21, 19, 17, 15, 13, ___,...
- R 39, 26, 13, ___,...
- T 48, 44, 40, 36, ___,...
- L 15, 18, 24, 33, ___,...
- S 378, 126, 42, ___,...
- E 72, 36, 18, ___,...
- S 17, 22, 27, 32, ___,...

Problem solving using algebra

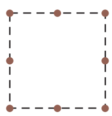
Finding a pattern or rule can be a very useful problem-solving technique. Once you have found the rule or formula, substitution allows you to use the same rule with many different input numbers. An example is shown below.

WORKED Example 9

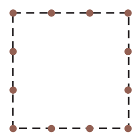
A farmer constructs temporary sheep pens by making a square, as shown in the diagrams below. A wire fence is attached to the posts which are placed at regular intervals around the pen.



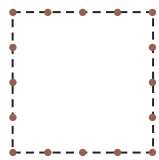
Number of posts on each side = 2
Total number of posts = 4



Number of posts on each side = 3
Total number of posts = 8



Number of posts on each side = 4
Total number of posts = 12



Number of posts on each side = 5
Total number of posts = 16

How many posts will be required for a square pen which has 45 posts on each side? (*Hint*: The answer is not 4×45 or 180 posts!)

THINK

1. Draw a table which shows the two sets of numbers and choose a pronumeral to label each column. We can call the number of posts on each side L and the total number of posts T.
2. Use trial and error to find a rule (formula) connecting L and T. For example, try multiplying each L value by 4 and then subtracting 4.
3. Write down the rule or formula.
4. To find the total number of posts, T, we must substitute the value 45 for L.

WRITE

Let L represent the number of posts on each side and T represent the total number of posts.

L	2	3	4	5
T	4	8	12	16

$4 \times 2 - 4 = 4$
$4 \times 3 - 4 = 8$
$4 \times 4 - 4 = 12$
$4 \times 5 - 4 = 16$

$T = 4L - 4$

If $L = 45$
$T = 4 \times 45 - 4$
$T = 180 - 4$
$T = 176$

This means that 176 posts are required for a pen with 45 posts on each side.

Notice in the example above that once we have found the formula which connects the two quantities, we can use it to find the number of posts that would be needed for a square pen of any size. For example, by substituting 20 for L, we could find the number of posts needed for a pen with 20 posts on each side. Try it.

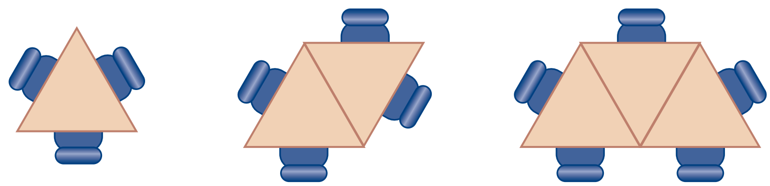
remember

1. Identify what your chosen pronumeral represents.
2. Once you have found a rule or formula, substitution allows you to use the same rule with many different input numbers.

EXERCISE 7D Problem solving using algebra

WORKED Example 9

1 The dining tables at the ‘Trio’ theatre restaurant are triangular in shape. Diners are seated at the tables in the arrangements shown below.



One table, 3 diners — Two tables, 4 diners — Three tables, 5 diners

The manager of the restaurant has received a booking from a large party. She sets out a row of 24 tables. How many diners can be seated at this row of tables?

Follow the steps below to solve this problem.

a Draw up a table with a column for the number of tables (T) and another column for the number of diners (D). Using the diagrams above, write in the pairs of numbers.

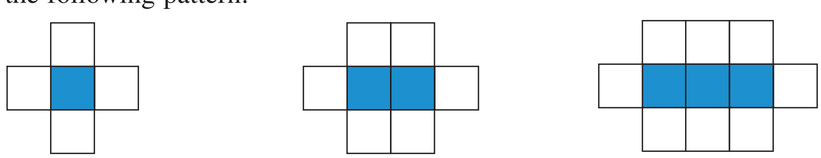
b Draw a diagram to work out the number of diners who could sit at a row of 4 tables. Now draw another diagram to show the number of diners who could sit at a row of 5 tables. Write these pairs of values in your table.

c Work out the formula which connects the number of diners and the number of tables. Write the formula in the form $D = \ldots$

d Substitute $T = 24$ into the formula to find out how many guests could be seated at the row of 24 tables.

SkillSHEET 7.5
GC program—Casio Finding the formula
GC program—TI Finding the formula
EXCEL Spreadsheet Substitution
Mathcad Substitution

2 Jane is tiling the floor in her bathroom. She has decided to use blue and white tiles in the following pattern:



Number of blue tiles = 1, Number of white tiles = 4 — Number of blue tiles = 2, Number of white tiles = 6 — Number of blue tiles = 3, Number of white tiles = 8

Find out how many white tiles Jane would need to complete the pattern for a row of 10 blue tiles by following the steps below:

a Draw up a table with two columns; one headed B showing the number of blue tiles and one headed W showing the number of white tiles required. Count the numbers of blue and white tiles from each of the tiling patterns above and put them into the table in the correct column.

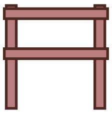
b Draw a diagram of the tiling pattern for a row of 4 blue tiles and work out the number of white tiles needed to complete the pattern. Then draw another pattern for a row of 5 blue tiles. Add these two sets of values for B and W into the table.

c Look carefully at the table and work out the formula which connects W and B. Write the formula in the form $W = \ldots$

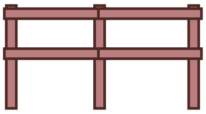
d Substitute $B = 10$ into the formula to find out how many white tiles would be needed for a row of 10 blue tiles, using Jane’s tiling pattern.

To answer questions **3** to **6**, use steps similar to those you used in questions **1** and **2**.

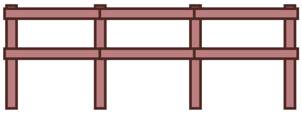
3 Michael is constructing a timber fence at his stud farm. The sections of fence are shown below:



1 section, 4 pieces



2 sections, 7 pieces



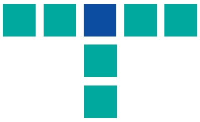
3 sections, 10 pieces

Michael calculates that he will need 220 sections to fence off his first paddock.

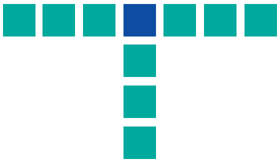
a Draw up a table which shows the number of pieces for each section.
b Write the formula for number of pieces, P.
c How many pieces of timber will Michael need?
d Find the cost of the fence if each piece of timber costs $3.85.

4 A T-shirt shop uses the letter T as its logo. The company wishes to make up a large advertising sign from square lights. Several small T-shirt shop signs are shown below:

Arm length = 1
Number of lights = 4



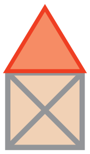
Arm length = 2
Number of lights = 7



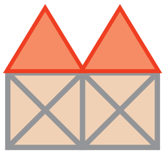
Arm length = 3
Number of lights = 10

Find out how many square light panels would be needed to make up a large sign with an arm length of 75.

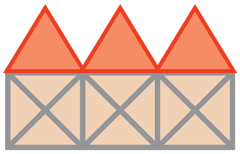
5 'Surfside' beach houses are constructed using metal framing struts. The diagrams below show the number of struts needed for the back wall of the beach houses, which are built in rows.



1 house, 8 struts



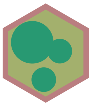
2 houses, 15 struts



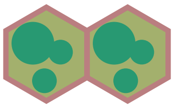
3 houses, 22 struts

'Surfside' has a contract to build a row of 34 beach houses at Golden Beach resort. How many metal struts will be required for the back wall of this row of beach houses?

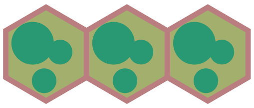
6 Johanna (a landscape gardener) is installing hexagonal flowerbeds in a park. The flowerbeds, which are surrounded by wooden sleepers, are arranged in rows as shown:



1 bed, 6 sleepers required

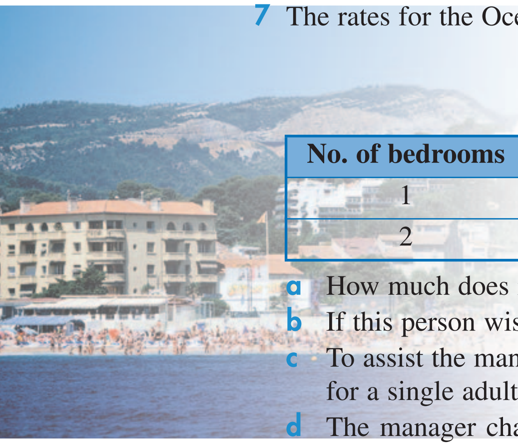


2 beds, 11 sleepers required



3 beds, 16 sleepers required

How many wooden sleepers will Johanna need to install a row of 12 flowerbeds?



7 The rates for the Ocean Plaza Resort are shown below.

	Cost per person ($), April–December			
	5 nights		Extra night	
No. of bedrooms	**Adult**	**Child***	**Adult**	**Child***
1	235	N/A	47	N/A
2	285	Free	57	Free

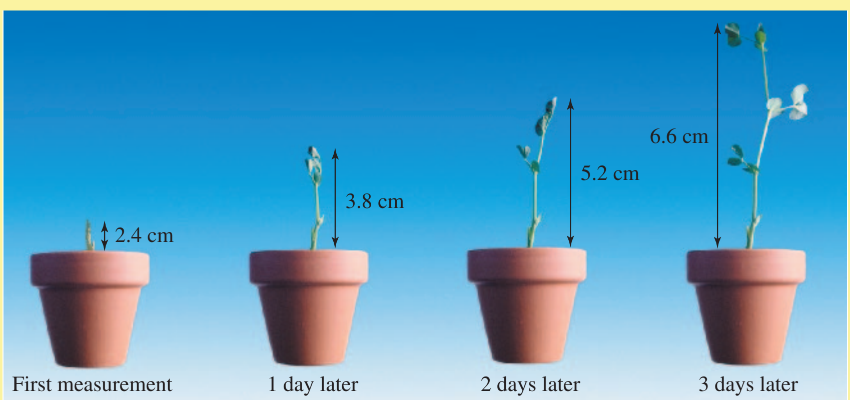
* Children aged 3–14 years

a How much does it cost for a single adult to stay 5 nights at the resort?
b If this person wishes to stay an extra 2 nights, what would the total charge be?
c To assist the manager in calculating charges, write a general rule to find the cost, c, for a single adult booking 5 nights then deciding to stay on for n extra nights.
d The manager charged $423 for a single adult who stayed from September 4 until checkout at 10 am on September 13. Is this correct?
e Three adults have booked a 2-bedroom suite for 5 nights with the option of staying extra nights. If they stay 2 extra nights, calculate the total charge.
f Write a general rule for the total charge, c, if the 3 adults stayed on for n extra nights.
g Use this rule to calculate the charge if the 3 adults stayed on for an extra 6 nights.

WorkSHEET 7.2

How high will it grow?

A seed you have planted in a pot is starting to grow. When you first notice the seedling, you measure it to be 2.4 centimetres high. Each day after this you continue to measure its height.



1 If the seedling continues to grow at the same rate, how tall will it be 4 days later?
2 How tall will it be 5 days later?
3 Write a formula connecting the height of the plant with the number of days after the plant height was first measured. Remember to identify what each pronumeral represents in your formula.
4 Check your formula by calculating the height of the plant after 7 days.
5 What is the height of the plant after 18 days?
6 What would be the height of the plant after 25 days?
7 Challenge: Use your formula to estimate when the plant would be approximately 32 centimetres high.
8 Give reasons why this formula may not be accurate to predict the plant height after a period of time.

10 Quick Questions 2

1 Rewrite $c = 6 \times k + 3 \times t$ without multiplication signs.

2 Insert multiplication signs wherever possible in $r = 17y + 8h - 2m$.

3 Complete the rule for the table by inserting a number in the space provided.

y	3	8	15	22	46	62
w	5	15	29	43	91	123

$w = 2y -$ ___

4 Study the following table and use the pronumerals to write a formula.

f	2	6	8	15	19	23
c	6	18	24	45	57	69

5 Use the rule given to complete the table.

$h = n + 5$

n	5	1	18	23	16	9
h						

6 Use the rule given to complete the table.

$c = 4r + 12$

r	8	2	15	41	32	16
c						

7 Substitute $k = 4$ into the formula $g = 5(k + 2)$ to find the value of g.

8 Substitute $t = 6$ into the formula $m = 4t - 3$ to find the value of m.

9 Substitute $f = 11$ into the formula $d = 5(2f - 7)$ to find the value of d.

10 If $k = 3$, $w = 5$ and $y = 7$, find the value of $2y - w + 4k$.

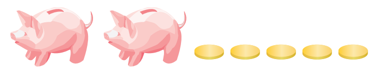
Algebraic expressions

We have already seen how we use pronumerals with rules and formulae. However, we also use pronumerals in *expressions*, which are part of a rule. Expressions are made up of pronumerals and numbers. For example, in the rule $y = 3x - 8$, we call '$3x - 8$' an expression. Some other examples of expressions include:

$$3t - 4, \quad 7 + 2k, \quad 3r + 6t, \quad 4y + 2y, \quad 6(g + 5), \quad 9p - 3y + 5.$$

When we want to solve problems with quantities that may vary, we can start building an expression by representing the variables with a pronumeral. Let's look at an example.

Molly and Vinh had a pile of 20-cent coins and some moneyboxes. They filled one moneybox before starting the next. When they had finished, they had two full moneyboxes and five coins left over.



To work out how many coins Molly and Vinh had, we would need to find out how many coins fitted into a moneybox. To help us write down an expression for this problem, we use a pronumeral to represent the number of coins in each moneybox.

Let c represent the number of coins in a moneybox.

We have two moneyboxes plus 5 coins = 2 lots of c plus 5 coins

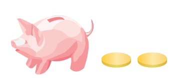
$= 2 \times c + 5$

$= 2c + 5$

Therefore, the number of coins Molly and Vinh had is $2c + 5$ coins.

WORKED Example 10

Write expressions to represent the total number of coins in each of the following situations where  **represents a full moneybox and** **represents one coin. Use c to represent the number of coins in a moneybox.**

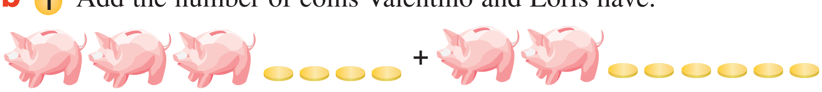
a **Mary fills one moneybox and has two coins left over.** 

b **Valentino and Loris decide to combine their coins. Valentino has three moneyboxes with four coins left over and Loris has two moneyboxes with six coins left over. That is,**

Valentino has 

Loris has 

THINK	WRITE
a 1 The total number of coins will be found by adding the number of coins in one moneybox plus two coins left over.	**a** $1 \times c + 2$
2 Remove the multiplication sign to write the expression.	$c + 2$
b 1 Add the number of coins Valentino and Loris have.	**b** $(3c + 4) + (2c + 6)$



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THINK	**WRITE**
2 Count the moneyboxes first, and then the coins.	$(3c + 2c) + (4 + 6)$

3 Simplify.	$5c + 10$

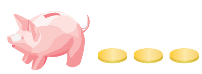
remember

1. Expressions are made up of pronumerals and numbers.
2. Pronumerals are used to represent quantities that vary, known as *variables*.

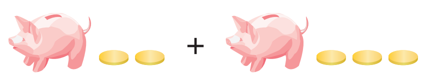
EXERCISE 7E Algebraic expressions

WORKED Example 10a

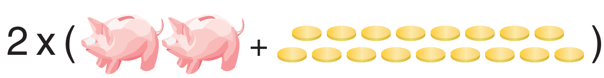
1 Write an expression for the total number of coins in each of the following, using c to represent the number of coins in a moneybox.

a  **b**  **c** 

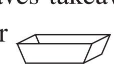
WORKED Example 10b

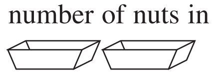
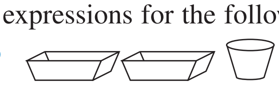
d  **e** 

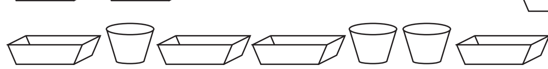
2 Christie and Jane both had two full moneyboxes and seventeen coins each. They combined their money and spent every cent on a day out in town.



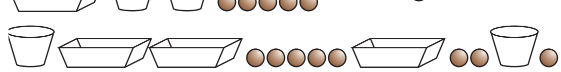
a Write an expression to represent the number of coins they had in total.
b Christie opened the first moneybox and counted 52 twenty-cent pieces inside. Use your expression to calculate how many coins the girls had in total, assuming that all the moneyboxes held the same number of coins.
c What is the total amount that the girls spent on their day out?

3 Luke has three macadamia nut trees in his backyard. He saves takeaway containers to store the nuts in. He has two types of containers, rectangular  and round .
Using m to represent the number of nuts in a rectangular container and n to represent the number of nuts in a round container, write expressions for the following.

a  **b** 

c 

d  where  represents one nut.

e 

4 Luke found that a rectangular container holds 17 nuts and a round container holds 12 nuts. Calculate how many nuts Luke would have for each part in question **3** by substituting the appropriate values into each expression.

Expressions and equations

We found in the previous section that *expressions* are parts of a rule made up of pronumerals and numbers.

Equations always contain an equals sign, whereas expressions do not. Here are some examples of equations:

$$y = 4t - 6, \quad m + 3 = y - 8, \quad 2(g + 5) = 5, \quad y = 6t, \quad w = e - 87, \quad 9 = 7 + 4r.$$

Expressions, and the equations shown above, are made up of *terms*. Terms may contain one or more pronumerals, such as $6t$ or $5axy$ or they may consist of a number only. Here are some examples of terms: $3t$, $2y$, 7, $5gh$, m, brt.

Information written in words can often be displayed more clearly as an expression. Look at the examples below.

WORKED Example 11

Write an expression for the sum of T and G.

THINK	WRITE
The word 'sum' means to add together.	The expression is $T + G$ or $G + T$.

When writing expressions, think about which operations are being used and the order in which they occur.

WORKED Example 12

If Y represents any number, write expressions for:

a **5 times that number**
b **2 less than that number**
c **8 more than that number**
d **the number divided by 4 (or the quotient of Y and 4)**
e **the next consecutive number (that is, the counting number which comes after Y).**

THINK	WRITE
In each case think about which operations are being used and the order in which they occur.	
a When multiplying we don't show the multiplication sign. Remember to put the number first.	**a** $5 \times Y$ or $5Y$
b An expression with 2 less means 'subtract 2'.	**b** $Y - 2$

Continued over page

THINK	**WRITE**
c An expression with 8 more means 'add 8'.	**c** $Y + 8$
d A number divided by 4 means 'write as a quotient (fraction)'.	**d** $\frac{Y}{4}$ or $Y \div 4$
e The next consecutive number means 'add 1 to the number'.	**e** $Y + 1$

WORKED Example 13

Write expressions for the following rules.

a Take a number and add another number.

b Multiply 2 numbers.

c Add 2 numbers and multiply the answer by 6.

d Take a number and multiply it by 4, then subtract 8 from that answer.

THINK	**WRITE**
1 In each case choose any pronumeral to represent an unknown number.	
2 In each case think about which operations are being used and the order in which they occur.	
a Add two pronumerals.	**a** $B + G$ or $G + B$
b Multiply two pronumerals but don't write a multiplication sign (×).	**b** xy
c Add two pronumerals and place the expression in brackets. Multiply by 6. (Remember to write the numbers first.)	**c** $6(t + f)$
d First multiply a pronumeral by 4, writing the number first ($4b$). Then subtract 8.	**d** $4b - 8$

remember

1. *Expressions* are parts of a rule made up of pronumerals and numbers.
2. *Equations* always contain an equals sign whereas expressions do not.
3. Expressions and equations are made up of *terms*.
4. When writing expressions, think about which operations are being used, and the order in which they occur.
5. If pronumerals are not given in a question, you can choose any letters you like.

EXERCISE 7F Expressions and equations

WORKED Example 11

SkillSHEET 7.6

1 Write an expression for each of the following.

a The sum of B and 2
b 3 less than T
c 6 added to D
d 5 taken away from K
e The sum of G, N and W
f D increased by H
g N increased by N
h H added to C
i G subtracted from 12
j D multiplied by 4
k 6 added to H
l The difference between Z and G
m B multiplied by F
n Y added to the product of 3 and M

2 Answer true or false for each of the statements below.

a $3x$ is a term.
b $3mn$ is a term.
c $g = 23 - t$ is an expression.
d $g = 5t - 6$ is an equation.
e $rt = r \times t$
f $5 + A = A + 5$
g $3d + 5$ is an expression.
h $7 - B = B - 7$
i $3x = 9$ is an expression.
j The expression g + $2t$ has two terms.
k $2f + 4 = 4 + 2f$
l $5a - 7t + h$ is an equation.

3 **multiple choice**

In each of the following, the letter N has been used to represent a number. For each expression written in words, choose the answer that you think matches:

a five times the number
A $N + 5$ B $5N$ C $N - 5$ D $N \div 5$

b the sum of the number and 52
A $52 - N$ B $52N$ C $N + 52$ D $N - 52$

c the next consecutive number
A O B $2N$ C $N - 1$ D $N + 1$

d the number multiplied by another number
A $N + T$ B $N \div S$ C $N + 4$ D NS

e half the number
A $2N$ B $N \div 2$ C $N + \frac{1}{2}$ D $N - 2$

f four more than the number
A $N + 4$ B $4N$ C $N - 4$ D $4 - N$

g the number plus the product of another number and 2
A $N + 2D$ B $D\,(N + 2)$ C $2N + 2B$ D $2N + D$

WORKED Example 12

4 If A, B and C represent any 3 numbers, write an expression for each of the following.

a The sum of all 3 numbers
b The difference between A and C
c The product of A and B
d The product of all 3 numbers
e The quotient of B and C (that is, B divided by C)
f The sum of A and C, divided by B
g 3 more than A

WORKED Example 13

5 Write expressions for the following rules.

a The number of students left in the class if X students leave for the canteen out of a total group of T students

b The amount of money earned by selling B cakes where each cake is sold for $4.00

c The total number of sweets if there are G bags of sweets with 45 sweets in each bag

d The cost of one concert ticket if 5 tickets cost $$T$

e The number of students in a class that contains R boys and M girls

f The number of insects if there are Y legs altogether

g The cost of M movie tickets if each ticket costs $11.00

h The total amount of fruit sold if A apples and H bananas are sold

i The number of cards left in a pack of 52 cards if J cards are removed

j The total number of floor tiles if there are B boxes of tiles with 12 tiles in each box

k The average of B and G

l The total runs scored by 3 batsmen if they have scores of A, H and K respectively

m The total number of legs if there are R chairs

MATHS QUEST CHALLENGE

1 If a mother is 37 years old and her daughter is 9 years old, in how many years will the mother be three times as old as the daughter?

2 A class of 25 students has 7 more boys than girls. How many boys are there?

3 Two men and two boys wish to cross a river from one side to the other. Their small canoe will carry only one man or two boys. What is the minimum number of canoe trips needed to get everyone across? List the number of people transported during each canoe trip and the destination.

What was the 'Model T' in 1908?

Match up the everyday situations with the appropriate formula. The number and letter beside each give the puzzle answer code.

- **S** Fish and chips cost \$2.80 per serve.
- **F** At a hotel, each room can accommodate 4 people.
- **U** Each person at the grand final pays a \$12 entry fee.
- **N** Mrs Min buys 5 eggs for each member of her family.
- **M** Table tennis bats cost \$18 each and balls are worth \$6 per person.
- **A** To go to the movies costs \$7 per person.
- **Y** Mark takes 6 minutes to wash each window of the house.
- **C** 'Bo' eats 3 cans of dog food each week.
- **H** Sally was paid \$15 per hour for a week.
- **O** A can of paint will cover an area of 16 square metres.
- **P** Sausages cost \$5/kg and chops cost \$6/kg.
- **D** A \$50 prize is to be split evenly between the members of the winning family.
- **I** Petrol for the car is worth 82 cents per litre.
- **R** Yvonne has 2 sandwiches per day on her holidays.
- **E** Ben's pocket money consists of \$10 plus \$1 for each time he has washed his dad's car.
- **T** Rolls cost 28 cents each.

1. $P = 15H$
2. $P = 10 + C$
3. $E = 5N$
4. $S = 2D$
5. $T = 6W$
6. $P = 4R$
7. $C = \frac{A}{16}$
8. $A = \frac{50}{N}$
9. $C = 82L$
10. $C = 28R$
11. $A = 12N$
12. $C = 3W$
13. $C = 2.8N$
14. $C = 7N$
15. $C = 5S + 6C$
16. $C = 18B + 6D$

1	2	3	4	5	6	7	4	8

9	3	10	4	7	8	11	12	2	8	10	1	2

6	9	4	13	10	12	1	2	14	15	16	14	13	13

15	4	7	8	11	12	2	8	12	14	4

Like terms

Sometimes it is possible to simplify an expression. Some commonsense rules are used to decide whether terms can be added or subtracted to make an expression simpler. In an expression like $6a + 3a$ the pronumeral 'a' represents the same number. This expression would be written as 'multiply an unknown number by 6, then multiply the same number by 3 and add the two answers together'.

If we wanted to show two different numbers, different pronumerals would be used. For example, the expression $6a + 3c$ would be written as 'multiply an unknown number by 6, then multiply another (different) unknown number by 3 and add the answers together'. The two terms $6a$ and $3a$ are called *like terms*, because the pronumeral represents the same number. On the other hand, $6a$ and $3c$ are not like terms.

When dealing with expressions that contain more than one pronumeral, such as $2ab + 3db$, the same rule applies; that is, like terms must contain exactly the same pronumerals. Remember that the order in which they are written does not matter, so $3ab$ and $7ba$ are like terms. Some examples are given below.

$5e$ and $6e$ are like terms (because the pronumeral is the same).
a and $5a$ are like terms (because the pronumeral is the same).
$3d$ and $6g$ are not like terms (because the pronumerals are different).
$8ab$ and $5ab$ are like terms.
$5ab$ and $4ba$ are like terms.
$4ab$ and $5a$ are not like terms.
$8ast$ and $7ast$ are like terms.
$3abc$ and $4bca$ are like terms.

Simplifying like terms

Look at the expression $5a + 3a$. Let's substitute some values of a into the expression:
If $a = 2$, $5a + 3a = 5 \times 2 + 3 \times 2 = 10 + 6 = 16$
If $a = 4$, $5a + 3a = 5 \times 4 + 3 \times 4 = 20 + 12 = 32$
If $a = 7$, $5a + 3a = 5 \times 7 + 3 \times 7 = 35 + 21 = 56$
Can you see that the result would be the same as if we had calculated the value of $8a$ in each case? Check this, as shown below:
if $a = 2$, $8a = 8 \times 2 = 16$; if $a = 4$, $8a = 8 \times 4 = 32$; if $a = 7$, $8a = 8 \times 7 = 56$.
This shows that $5a + 3a = 8a$.

The process of adding or subtracting like terms is called *simplifying*; that is, writing the expression in a more simple form.
Here are some more examples of simplifying:

$6b + 4b = 10b$
$5xy + 8xy = 13xy$
$12y + 3y - 5y = 10y$
$4t - 3t = t$
$7t - 5t = 2t$

Remember that simplifying can be done only with like terms. (The pronumerals must be exactly the same.) For example:

$5g + 5c$ cannot be simplified
$4t - 4$ cannot be simplified
$3xy + 4x$ cannot be simplified
$14abc - 6bc$ cannot be simplified
$45b - 43$ cannot be simplified
$4g - 4n$ cannot be simplified.

WORKED Example 14

Where possible, simplify the following expressions by adding or subtracting like terms.

a $4g + 6g$ **b** $11ab - ab$ **c** $6ad + 5da$ **d** $4t + 7t - 5$ **e** $8x + 3y$

THINK	WRITE
a The pronumerals are the same. The terms can be added.	**a** $4g + 6g = 10g$
b The pronumerals are the same. The terms can be subtracted. Note that *ab* is the same as 1*ab*.	**b** $11ab - ab = 10ab$
c Although the order of the pronumerals is different, the terms are like terms and can be added.	**c** $6ad + 5da = 11ad$ (or $11da$)
d The pronumerals are the same and the first two terms can be added. Do not add the other term.	**d** $4t + 7t - 5 = 11t - 5$
e The pronumerals are not the same. These are not like terms and therefore cannot be simplified.	**e** $8x + 3y$ cannot be simplified.

In the next worked example we will see that substituting known values for pronumerals in an expression allows us to evaluate the expression.

WORKED Example 15

Simplify the following expressions first, then find the value of the expression if $a = 4$.

a $5a + 2a$ **b** $7a - a + 5$

THINK	WRITE
a 1 Add the like terms. 2 Substitute 4 for *a* in the simplified expression.	**a** $5a + 2a = 7a$ If $a = 4$ $7a = 7 \times a$ $= 7 \times 4$
3 Evaluate.	$= 28$
b 1 Subtract the like terms. 2 Substitute 4 for *a* in the simplified expression.	**b** $7a - a + 5 = 6a + 5$ If $a = 4$ $6a + 5 = 6 \times a + 5$ $= 6 \times 4 + 5$
3 Evaluate.	$= 29$

remember

1. Terms containing the same pronumerals are called *like terms*.
2. The process of adding or subtracting like terms is called *simplifying*.
3. Substituting known values for pronumerals in an expression allows us to evaluate the expression.

EXERCISE 7G Like terms

1 Answer True or False to each of the following statements.

- **a** $4t$ and $6t$ are like terms.
- **b** $3x$ and x are like terms.
- **c** $5g$ and $5t$ are like terms.
- **d** $7t$ and $5i$ are like terms.
- **e** $5a$ and $6a$ are like terms.
- **f** $4a$ and $4ab$ are like terms.
- **g** $9fg$ and $4gf$ are like terms.
- **h** $6gh$ and $7gk$ are like terms.
- **i** $6r$ and $5r$ are like terms.
- **j** yz and $45zy$ are like terms.
- **k** $3acd$ and $6cda$ are like terms.
- **l** $5g$ and $5fg$ are like terms.
- **m** $8gefh$ and $3efgh$ are like terms.
- **n** $6ab$ and $3ba$ are like terms.
- **o** $8xy$ and $5xy$ are like terms.
- **p** $12ep$ and $4pe$ are like terms.
- **q** $7eg$ and $7g$ are like terms.
- **r** $7y$ and $18yz$ are like terms.

2 **multiple choice**

GAME time Algebra — 001

- **a** Which one of the following is a like term for $7t$?
  A $7g$ **B** $5t$ **C** 2 **D** $7tu$
- **b** Which one of the following is a like term for $3a$?
  A $5ab$ **B** $6a$ **C** $3w$ **D** $a + 2$
- **c** Which one of the following is a like term for $5ab$?
  A $6abe$ **B** $4b$ **C** $2ba$ **D** 5
- **d** Which one of the following is a like term for $7bgt$?
  A $7t$ **B** $4gt$ **C** $2gb$ **D** btg
- **e** Which one of the following is a like term for $2fgk$?
  A $fg + k$ **B** $5fg$ **C** $2fgk$ **D** $12f$
- **f** Which one of the following is a like term for $20m$?
  A $11m$ **B** $20mn$ **C** $20f$ **D** $m + 2$
- **g** Which one of the following is a like term for $9xyz$?
  A $9x$ **B** $2yz$ **C** $8xz$ **D** xzy

3 Answer True or False to each of the following statements.

- **a** The equation $y = 4x + 3x$ can be simplified to $y = 7x$.
- **b** The equation $k = 8y + 4y$ can be simplified to $k = 10y$.
- **c** The equation $y = 4x + 3$ can be simplified to $y = 7x$.
- **d** The equation $b = 3a - a$ can be simplified to $b = 2a$.
- **e** The equation $k = 7a + 4d$ can be simplified to $k = 11ad$.
- **f** The equation $y = 5x - 3x$ can be simplified to $y = 2x$.
- **g** The equation $m = 7 + 2x$ can be simplified to $m = 9x$.
- **h** The equation $p = 3x - 2x$ can be simplified to $p = x$.
- **i** The equation $t = 3h + 12h + 7$ can be simplified to $t = 15h + 7$.
- **j** The equation $y = 16g + 6g - 7g$ can be simplified to $y = 15g$.

WORKED Example 14

4 Where possible, simplify the following expressions by adding or subtracting like terms.

Mathcad: Simplifying expressions

a $3a + 2a$ **b** $9y + 5y$ **c** $3c + 12c$
d $4u - 2u$ **e** $7e + 13e$ **f** $7t - 2t$
g $12ab + 2ab$ **h** $8fg - 2fg$ **i** $4e - e$
j $6t + t$ **k** $f + 4f$ **l** $6y - 6y$
m $4x + 14x$ **n** $3m + 16m - 7m$ **o** $6a + 4a - 2a$

5 Simplify the following expressions if possible.

a $24ab + ab - 7$ **b** $5y + y - 3y$ **c** $5t + 5s$
d $18i + 12i - 2$ **e** $4t + 8t - 3 + 2t$ **f** $7r + 2r + 5r - r$
g $2x + 2y$ **h** $7y + 6$ **i** $18f - 2f + 5$
j $4 + 3g - g$ **k** $6t - 5t$ **l** $18bg - 18bg$
m $13mno - 11mno$ **n** $11pq + 3qp$ **o** $6pr + 2 + 5rp$
p $9e - 9t - 1$ **q** $7t + 4t - 5$ **r** $32t - 31t$
s $11aw - 3aw$ **t** $7xy - 7x$ **u** $5t + 6t - 8$
v $5t - t + 3$ **w** $3g + 7g - 2$ **x** $5r + 17r + 4 + 2$

6 Simplify:

GAME time: Algebra — 002

a $3t - 3t$ **b** $18r - 18r$ **c** $12ab - 12ab$
d $5x - 5x + 8$ **e** $6t + 7 - 6t$ **f** $9g - 9g + 2$
g $13xyz - 13xyz$ **h** $5x + 7 - 5x$ **i** $5y + 2y - y$

WORKED Example 15

7 Simplify the following expressions first, then find the value of the expression if $a = 7$:

WorkSHEET 7.3

a $3a + 2a$ **b** $7a + 2a$ **c** $6a - 2a$
d $9a + a$ **e** $13a + 2a - 5a$ **f** $3a + 7a$
g $17 + 5a + 3a$ **h** $6a - a + 2$ **i** $a + a$
j $7a - 6a$ **k** $7a - 7a$ **l** $12a + 5a - 16$

MATHS QUEST CHALLENGE

A *cryptarithm* is a puzzle where each digit of a number has been replaced by a pronumeral or letter. Solve the following cryptarithms by determining the digit that corresponds to each letter to make the calculation true.

1
$$\begin{array}{r} \text{A B} \\ +\ \text{B A} \\ \hline \text{C A C} \\ \hline \end{array}$$

2
$$\begin{array}{r} \text{D} \\ \times\ \text{D} \\ \hline \text{E D} \\ \hline \end{array}$$

3 **This cryptarithm has several solutions. Find at least 4 solutions.**
$$\begin{array}{r} \text{T W O} \\ +\ \text{T W O} \\ \hline \text{F O U R} \\ \hline \end{array}$$

4
$$\begin{array}{r} \text{S E N D} \\ +\ \text{M O R E} \\ \hline \text{M O N E Y} \\ \hline \end{array}$$

5
$$\begin{array}{r} \text{O N E} \\ +\ \text{T W O} \\ +\ \text{F O U R} \\ \hline \text{S E V E N} \\ \hline \end{array}$$

summary

Copy the sentences below. Fill in the gaps by choosing the correct word or expression from the word list that follows.

1 In algebra, a rule can be used to produce a set of __________ numbers from a set of input numbers.

2 A letter or symbol which is used in place of a number is called a __________.

3 The statement $m = b + 2$, which shows the way in which two sets of numbers (m and b) are related is called a __________ or __________.

4 When __________ numbers and pronumerals we can leave out the multiplication sign.

5 We place numbers in __________ of pronumerals when multiplying.

6 Replacing a pronumeral with a number is called __________.

7 The expression $8y - 4$ contains two __________.

8 Equations are different from expressions because equations always include an __________ sign.

9 Like terms contain the same __________.

10 The process of adding together like terms is called __________.

11 The __________ $3x + 4y - 2x$ contains two __________.

12 Substituting known values for pronumerals in an expression allows us to __________ the expression.

WORD LIST

multiplying	expression	pronumeral	terms
equals	formula	simplifying	pronumerals
equation	like terms	evaluate	
front	output	substitution	

CHAPTER review

1 Copy and complete the tables below. For each table, use the rule to work out the correct output numbers. 7A

a Multiply each input number by 4.

Input	3	1	13	7	4
Output					

b Add 7 to each input number.

Input	4	1	15	6	7
Output					

c Multiply each input number by 2, then subtract 3.

Input	3	10	13	7	4
Output					

d Divide each input number by 4.

Input	4	8	16	20	0
Output					

2 Copy and complete the tables below. Use the rule given in each case to work out the missing input and output numbers. 7B

a Subtract 7 from each input number.

Input	15	16	33		
Output				3	7

b Multiply each input number by 3, then add 7.

Input	4	1	15		
Output				22	106

c Add 5 to each input number, then multiply by 2.

Input	3	1	6		
Output				18	96

d Multiply each input number by 2, then subtract 5.

Input	15	8	11		
Output				21	71

3 **multiple choice** 7B

Which formula has the algebra machine used in each of the following tables?

a

a	1	5	4	23	6
g	11	15	14	33	16

A $g = 5a + 6$ **B** $g = a - 10$
C $g = a + 10$ **D** $g = 10a$

b

m	3	1	11	2	4
t	8	2	32	5	11

A $t = 3m$ **B** $t = 3m - 1$
C $t = m + 5$ **D** $t = 3m + 1$

4 Look carefully at the pattern in each of the following tables. Then complete the formula for each table by inserting a number into the gap. 7B

a

x	1	7	23	4	8
y	5	59	203	32	68

$y = 9x - \_\_\_$

b

p	3	6	12	5	0
q	15	24	42	21	6

$q = \_\_\_\, p + 6$

5 Use the pronumerals given to write a formula for each table. 7B

a

x	3	4	7	2	0
y	9	10	13	8	6

b

c	13	6	8	12	5
d	8	1	3	7	0

c

g	4	6	23	3	9
h	22	32	117	17	47

d

m	1	5	11	6	4
n	3	19	43	23	15

7C

6 Complete the tables below by substituting each of the input numbers (x) into the rule.

a $y = x - 1$

x	1	4	7	3	105
y					

b $d = 3x$

x	2	12	1	6	7
d					

c $h = 6 - x$

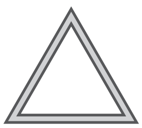
x	0	6	4	5	2
h					

d $n = 11x + 3$

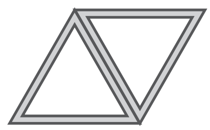
x	1	4	7	3	0
n					

7D

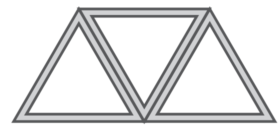
7 The metal ceiling rafters in a school classroom consist of a series of triangles connected end to end as shown below. The number of triangle side lengths required in each rafter depends on the total number of triangles. However, adjacent triangles share a common side length.



1 triangle, 3 side lengths



2 triangles, 5 side lengths



3 triangles, 7 side lengths

How many side lengths will be needed to build a single rafter if each rafter contains 25 triangles? Follow the steps below:

a Draw up a table with columns headed T (the number of triangles), and S (the number of side lengths needed). Write in the numbers from the diagrams above.

b Complete a diagram showing the number of side lengths needed for a rafter containing 4 triangles. Add these values to your table.

c Look at the table to determine the formula which connects S and T. Write the formula in the form $S = \ldots$

d Substitute $T = 25$ into the formula to find the number of side lengths needed for each ceiling rafter.

7E

8 Otto works in a warehouse, packing boxes of CDs for distribution to music stores. An order from Sanity's city store fills 35 boxes with 18 CDs left over.

a Write an expression to represent the Sanity order, using the pronumeral x to represent a full box of CDs.

b If each full box contains 30 CDs, use your expression to calculate how many CDs the store ordered.

7F

9 Write expressions for the following:

a the difference between M and C

b money earned by selling B cakes for \$3 each

c the product of X and Y

d 15 more than G

e 1 more than D

f the cost of 12 bananas at H cents each

g T multiplied by 5.

7G

10 Simplify the following expressions by adding or subtracting like terms.

a $3g + 4g$ **b** $8y - 2y$ **c** $4h + 5h$ **d** $7ag - 2ag$

e $6gy - 3yg$ **f** $8r - 8r$ **g** $6y - 2y + y$ **h** $4t + 6 + 3t$

i $12gh + 6hg$ **j** $8t - 2m + 3t$ **k** $3m + m$ **l** $7g + 8g + 8 + 4$

m $7h + 4t - 3h$ **n** $2b + 7c + 8b$ **o** $11axy - 3axy$

7G

11 Simplify the following expressions first, then find the value of the expression if $x = 5$.

a $7x + 3x$ **b** $2x + 3x - 4$ **c** $11x + 12x$

d $x + 2x$ **e** $4x - x$ **f** $3x - 2x + 16$

g $21x - 13x + 7$ **h** $11 + 2x + 5x$ **i** $7x - 4x + 3x$