5 Fractions

A pepperoni pizza is cut into 6 equal pieces while a sausage pizza of the same size is cut into 8 equal pieces. If Martin eats 2 pieces of the pepperoni pizza and Olivia eats 3 pieces of the sausage pizza, who has eaten more? How can you prove this?

In this chapter, you will learn to compare, order and calculate with fractions.

Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest 7* CD-ROM or ask your teacher for a copy.

SkillSHEET 5.1

1 State whether each of the following is a proper fraction, an improper fraction or a mixed numeral.

a $\frac{5}{6}$ **b** $\frac{9}{4}$ **c** $3\frac{1}{2}$

d $\frac{3}{2}$ **e** $1\frac{3}{4}$ **f** $\frac{11}{20}$

SkillSHEET 5.2

2 **a** Form an equivalent fraction to $\frac{2}{3}$ by multiplying the numerator and denominator by 5.

b Form an equivalent fraction to $\frac{24}{28}$ by dividing the numerator and denominator by 4.

SkillSHEET 5.3

3 Copy and complete the following equivalent fractions by filling in the gaps.

$\frac{1}{3} = \frac{2}{\_} = \frac{\_}{9} = \frac{\_}{12}$

SkillSHEET 5.4

4 **a** List the first eight multiples of 3.
b List the first eight multiples of 4.
c Which multiples are common (the same) in both lists?
d What is the lowest common multiple of 3 and 4?

SkillSHEET 5.5

5 Write these fractions in ascending order (from smallest to largest).

a $1, \frac{4}{5}, \frac{1}{5}, \frac{3}{5}, \frac{7}{5}$ **b** $\frac{11}{12}, 0, \frac{7}{12}, \frac{13}{12}, \frac{5}{12}$

SkillSHEET 5.6

6 **a** List the factors of 18.
b List the factors of 12.
c Which factors are common (the same) in both lists?
d What is the highest common factor of 18 and 12?

SkillSHEET 5.7

7 Use a diagram to express $\frac{5}{2}$ as a mixed numeral.

SkillSHEET 5.8

8 Use a diagram to express $1\frac{1}{3}$ as an improper fraction.

SkillSHEET 5.9

9 Find:

a $\frac{5}{6} + \frac{3}{6}$ **b** $\frac{7}{12} - \frac{5}{12}$

c $1 - \frac{1}{3}$ **d** $4 - \frac{1}{4}$

Understanding fractions

Antonio makes pizzas.

Besides selling whole pizzas, he often sells part of a pizza to his customers. For example, this pizza has been cut into three equal pieces. If a customer wants only two of these slices, then Antonio is selling $\frac{2}{3}$ of the pizza to the customer.

Fractions are used to describe parts of a whole. The *numerator*, or top number of the fraction, shows how many parts are required, while the *denominator* or bottom number shows the number of parts the whole has been divided into. So $\frac{2}{3}$ means that the whole thing has been divided into three equal parts and that we are concerned with two parts. The horizontal bar separating the numerator from the denominator is called the *vinculum*.

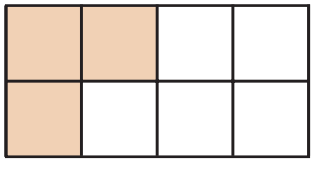
A *proper fraction* is a fraction with the numerator smaller than the denominator, like $\frac{2}{3}$.

An *improper fraction* has a numerator that is larger than the denominator, like $\frac{5}{4}$.

A *mixed numeral* is made up of a number and a fraction, like $1\frac{3}{5}$.

WORKED Example 1

What fraction of the rectangle shown has been shaded?



THINK

1. Count how many equal parts the rectangle has been divided into.
2. How many parts are shaded.
3. Write the answer in a sentence.

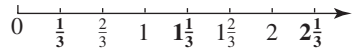
WRITE

Total number of parts = 8

3 parts are shaded.

$\frac{3}{8}$ of the rectangle has been shaded.

Fractions can be marked on a number line in the same way as counting numbers.

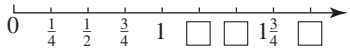


The number line above can be used to count in fractional steps. The fractions marked in bold are found by adding 1 to the previous fraction or mixed numeral so that the following sequence of numbers is obtained:

$$\frac{1}{3}, 1\frac{1}{3}, 2\frac{1}{3}, 3\frac{1}{3}, 4\frac{1}{3}, \ldots$$

WORKED Example 2

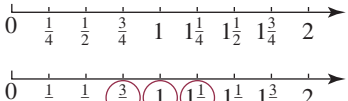
Copy the number line shown, filling in the missing numbers. Use it to find the next three terms in the given sequence: $\frac{3}{4}$, 1, $1\frac{1}{4}$, _, _, _.

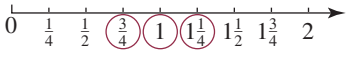


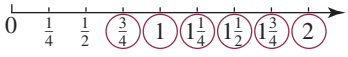
THINK

1. Copy the number line and fill in the missing numbers.
2. Circle the three given numbers in the sequence on the number line.
3. On the number line, circle the next three terms which continue the sequence.
4. Write the sequence with the missing numbers.

WRITE







$\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2

Finding equivalent fractions

Antonio cuts a pizza into two equal pieces as shown. Each piece is exactly half of the pizza.

A lady with a small child comes into the shop and wants to buy some pizza. She decides to cut one of the halves into two smaller, equal pieces as shown.

Now the child eats two quarters of a pizza and the mother eats half a pizza. If you look carefully you will see that each person has the same amount of pizza. We can say that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$ or $\frac{1}{2} = \frac{2}{4}$.

Equivalent fractions can be found by multiplying or dividing both the numerator and the denominator by the same number as long as the number is not zero. Therefore,

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4} \quad \text{and} \quad \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}.$$

$$\text{So} \quad \frac{1}{2} = \frac{2}{4} = \frac{3}{6}.$$

WORKED Example 3

For the equivalent fractions $\frac{4}{5} = \frac{8}{10}$ find the number which has been used to multiply both numerator and denominator.

THINK

1. What number is 4 multiplied by to equal 8? $(4 \times \_ = 8)$
   What number is 5 multiplied by to equal 10? $(5 \times \_ = 10)$
2. Write the answer.

WRITE

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

The number used is 2.

WORKED Example 4

From the list, find those fractions which are equivalent to $\frac{1}{2}$: $\frac{3}{9}$, $\frac{3}{6}$, $\frac{9}{18}$, $\frac{10}{15}$, $\frac{7}{14}$, $\frac{17}{34}$.

THINK

1. Multiply the numerator and denominator of $\frac{1}{2}$ by the numerator of the first fraction (3) to check whether the new fraction is in the list.
2. Multiply the numerator and denominator of $\frac{1}{2}$ by the next different numerator (9) and check whether the new fraction is in the list.
3. Continue until all fractions have been considered.
4. Write the equivalent fractions.

WRITE

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1 \times 9}{2 \times 9} = \frac{9}{18}$$

$$\frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

$$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

$$\frac{1 \times 17}{2 \times 17} = \frac{17}{34}$$

Equivalent fractions of $\frac{1}{2}$ are $\frac{3}{6}$, $\frac{9}{18}$, $\frac{7}{14}$, $\frac{17}{34}$.

WORKED Example 5

Write three equivalent fractions for $\frac{2}{3}$.

THINK

1. Multiply both the numerator and denominator by 2, 3 and 4.
2. Write the equivalent fractions.

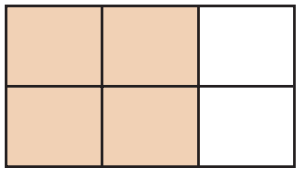
WRITE

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{2 \times 3}{3 \times 3} = \frac{2 \times 4}{3 \times 4}$$

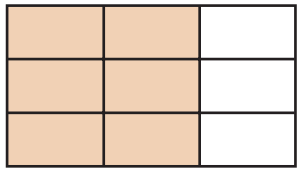
$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

Note: There are other fractions which are equivalent to $\frac{2}{3}$. These can be found by multiplying the numerator and denominator by other numbers. Answers to exercises (at the back of the book) will always give the smallest three equivalent fractions.

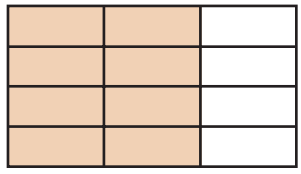
Equivalent fractions can also be shown using diagrams. Equivalent fractions for $\frac{2}{3}$ are shown below.



$\frac{2}{3} = \frac{4}{6}$



$\frac{2}{3} = \frac{6}{9}$



$\frac{2}{3} = \frac{8}{12}$

Notice that the same amount of the rectangle has been shaded in each case.

Comparing fractions

If Antonio's pizza is cut into quarters it would be easy to see that 1 piece is less than 2 pieces, or that $\frac{1}{4}$ of the pizza is less than $\frac{2}{4}$.

If Antonio has another pizza of the same size but cuts it into thirds, it can still be easily seen that $\frac{1}{4}$ is less than $\frac{1}{3}$.

This is because a pizza divided into four equal pieces will give smaller pieces than one divided into three equal pieces.

If the numerators are the same, the smaller fraction is the one with the larger denominator. For example, $\frac{1}{6}$ is greater than $\frac{1}{7}$ and $\frac{3}{10}$ is less than $\frac{3}{4}$. If the numerators are different, another method is required to compare fractions.

Fractions which do not have the same denominator can be compared by using equivalent fractions. We need to rewrite the fractions so that they have the same denominator.

To do this we look at the multiples of both denominators and choose the lowest multiple that is common to both.

For example, to compare the fractions $\frac{3}{4}$ and $\frac{2}{3}$, we need to find a number that can be used as the new denominator for each equivalent fraction. We start by listing the multiples of 4 and 3.

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, …

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, …

From these lists we can see that the lowest number that is the same in each is 12. (We can also see that 24 is the same, but it is not the lowest.) So the lowest common multiple (LCM) of 4 and 3 is 12. As 12 is divisible by both 4 and 3, each fraction can be written as an equivalent fraction using 12 as the denominator.

WORKED Example 6

Which is the bigger fraction, $\frac{2}{3}$ or $\frac{3}{5}$?

THINK

1. Find the lowest common multiple of the denominators. First list the multiples of 3 and 5. Identify the lowest number that is common to both lists.
2. Write each fraction as an equivalent fraction using this number (15) as the denominator.
3. Decide which is bigger by comparing the numerators of the equivalent fractions ($\frac{10}{15}$ and $\frac{9}{15}$).

WRITE

Multiples of 3 are 3, 6, 9, 12, 15, 18, …
Multiples of 5 are 5, 10, 15, 20, …
The lowest common multiple is 15.

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$ and $\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$

$\frac{2}{3}$ is bigger than $\frac{3}{5}$.

The answer to worked example 6 can be written as $\frac{2}{3} > \frac{3}{5}$. The symbol '>' means *is bigger than* or *is greater than*. The symbol '<' means *is less than*. For example, $\frac{1}{2} < \frac{3}{4}$ means that $\frac{1}{2}$ is less than $\frac{3}{4}$. An easy way to remember this is that < looks a little bit like the capital letter L (for less than).

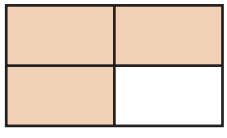
remember

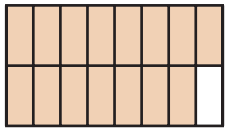
1. *Equivalent fractions* can be found by multiplying or dividing both the numerator and the denominator by the same non-zero number.
2. The *numerator* is the top part of the fraction.
3. The *denominator* is the bottom part of the fraction.
4. Different fractions can be compared or ordered by writing each one with the same denominator. This is called finding the lowest common multiple (LCM).
5. The symbol '>' means *is greater than*, the symbol '<' means *is less than*.

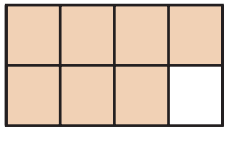
EXERCISE 5A Understanding fractions

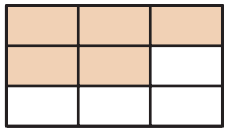
WORKED Example 1

1 What fraction of each of the following rectangles has been shaded?

a 

b 

c 

d 

SKILLSHEET 5.1

2 What fraction of each of these flags is coloured red?

a

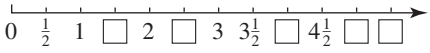
b

Do you recognise these flags? Can you name the country to which each flag belongs?

WORKED Example 2

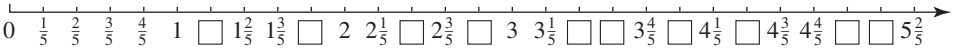
3 Copy the number lines shown, filling in the missing numbers. Then use them to find the next three terms in the given sequence.

a



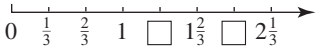
$\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \_\_, \_\_, \_\_$

b



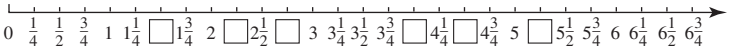
$\frac{1}{5}, 1\frac{1}{5}, 2\frac{1}{5}, \_\_, \_\_, \_\_$

c



$\frac{1}{3}, \frac{2}{3}, 1, \_\_, \_\_, \_\_$

d



$\frac{1}{4}, 1\frac{1}{2}, 2\frac{3}{4}, \_\_, \_\_, \_\_$

SkillSHEET 5.2

WORKED Example 3

4 For the equivalent fractions below find the number which has been used to multiply both numerator and denominator.

a $\frac{3}{8} = \frac{9}{24}$ **b** $\frac{1}{4} = \frac{3}{12}$ **c** $\frac{3}{5} = \frac{12}{20}$

d $\frac{2}{3} = \frac{10}{15}$ **e** $\frac{5}{6} = \frac{30}{36}$ **f** $\frac{9}{10} = \frac{81}{90}$

g $\frac{7}{8} = \frac{77}{88}$ **h** $\frac{7}{8} = \frac{84}{96}$ **i** $\frac{2}{5} = \frac{14}{35}$

EXCEL Spreadsheet Equivalent fractions

WORKED Example 4

5 From the list, find those fractions which are equivalent to the fractions marked in red.

a $\mathbf{\frac{2}{3}}$ $\frac{7}{9}$ $\frac{20}{30}$ $\frac{5}{8}$ $\frac{12}{16}$ $\frac{14}{21}$ $\frac{40}{60}$

b $\mathbf{\frac{4}{5}}$ $\frac{12}{15}$ $\frac{15}{20}$ $\frac{36}{45}$ $\frac{16}{20}$ $\frac{28}{35}$ $\frac{80}{100}$

c $\mathbf{\frac{7}{8}}$ $\frac{17}{18}$ $\frac{40}{45}$ $\frac{56}{64}$ $\frac{14}{18}$ $\frac{18}{19}$ $\frac{21}{24}$

d $\mathbf{\frac{7}{10}}$ $\frac{18}{25}$ $\frac{35}{50}$ $\frac{14}{21}$ $\frac{21}{30}$ $\frac{14}{20}$ $\frac{140}{200}$

EXCEL Spreadsheet Equivalent fractions (DIY)

SkillSHEET 5.3

6 Fill in the gaps.

a $\frac{3}{5} = \frac{9}{\_\_} = \frac{\_\_}{20} = \frac{\_\_}{35}$ **b** $\frac{3}{4} = \frac{\_\_}{40} = \frac{18}{\_\_} = \frac{\_\_}{36}$

c $\frac{1}{4} = \frac{\_\_}{28} = \frac{6}{\_\_} = \frac{3}{\_\_}$ **d** $\frac{5}{6} = \frac{15}{\_\_} = \frac{\_\_}{42} = \frac{40}{\_\_}$

WORKED Example 5

7 Write three equivalent fractions for each of the following.

a $\frac{5}{6}$ **b** $\frac{5}{8}$ **c** $\frac{3}{10}$ **d** $\frac{2}{3}$ **e** $\frac{1}{8}$ **f** $\frac{7}{8}$

WORKED Example 6

8 Which is the bigger fraction?

a $\frac{2}{5}$ or $\frac{3}{5}$ **b** $\frac{5}{8}$ or $\frac{7}{8}$ **c** $\frac{1}{5}$ or $\frac{1}{6}$ **d** $\frac{1}{8}$ or $\frac{1}{10}$ **e** $\frac{1}{2}$ or $\frac{3}{10}$

f $\frac{3}{4}$ or $\frac{3}{5}$ **g** $\frac{2}{5}$ or $\frac{5}{8}$ **h** $\frac{3}{5}$ or $\frac{5}{8}$ **i** $\frac{4}{5}$ or $\frac{5}{8}$

SkillSHEET 5.4

9 Insert the appropriate symbol, > or <, between each pair of fractions to make a true statement.

a $\frac{5}{8}$ $\frac{3}{8}$ **b** $\frac{5}{6}$ $\frac{7}{6}$ **c** $\frac{3}{5}$ $\frac{1}{4}$ **d** $\frac{3}{4}$ $\frac{3}{5}$ **e** $\frac{1}{5}$ $\frac{1}{4}$ **f** $\frac{3}{10}$ $\frac{2}{5}$

GC program – Casio: Comparing fractions

10 **multiple choice**

a Which fraction is smaller than $\frac{5}{8}$?

A $\frac{7}{8}$ **B** $\frac{17}{24}$ **C** $\frac{11}{12}$ **D** $\frac{3}{5}$

b Which fraction is equivalent to $\frac{3}{4}$?

A $\frac{3}{8}$ **B** $\frac{6}{8}$ **C** $\frac{9}{16}$ **D** $\frac{12}{24}$

GC program – TI: Comparing fractions

11 Write the following fractions with the same denominator and then write them in ascending order. (Ascending means from smallest to largest).

a $\frac{3}{10}, \frac{1}{2}, \frac{1}{5}$ **b** $\frac{3}{8}, \frac{1}{2}, \frac{1}{3}$ **c** $\frac{2}{3}, \frac{4}{5}, \frac{6}{15}$ **d** $\frac{3}{4}, \frac{2}{3}, \frac{7}{15}$

SkillSHEET 5.5

12 Four rather hungry teenage boys ordered pizzas, but none could eat the whole pizza.

Peter Pepperoni ate $\frac{6}{8}$ of his pepperoni pizza.

Harry Hawaiian ate $\frac{1}{2}$ of his Hawaiian pizza.

Sammy Special ate $\frac{3}{4}$ of his special pizza.

Vince Vegetarian ate $\frac{5}{6}$ of his vegetarian pizza.

a Draw the four pizzas and shade the amount that each boy ate.
b Which boy ate the most pizza?
c Who ate more pizza, Peter Pepperoni or Sammy Special?

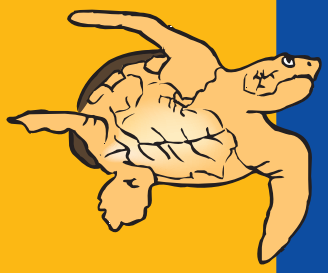
GAME time: Fractions — 001

MATHS QUEST CHALLENGE

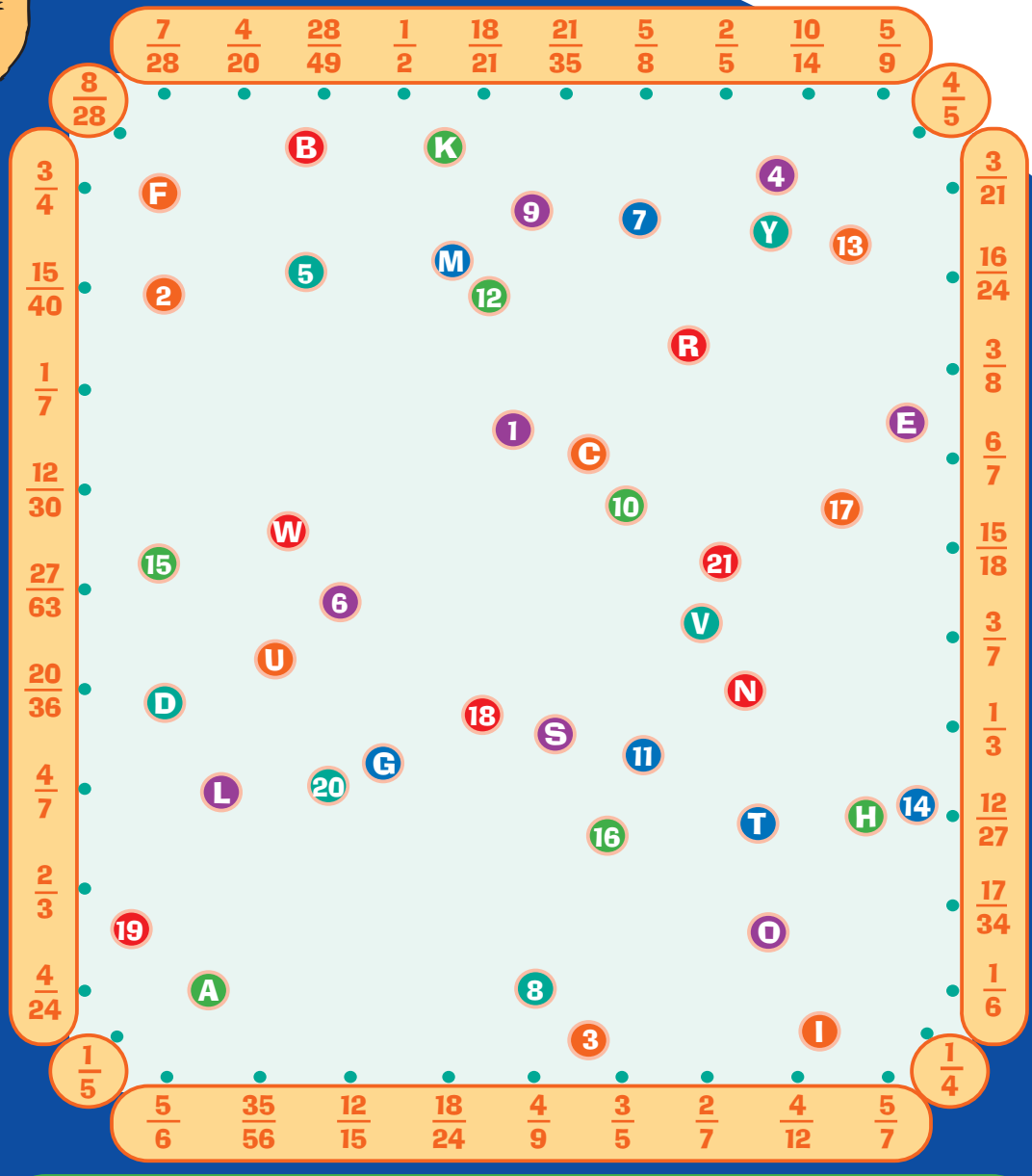
1 **Can you arrange the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to form a fraction which is equivalent to $\frac{1}{2}$? For example, one way is $\frac{7329}{14\,658}$. Can you find two others?**
***Hint*: Try starting with a numerator beginning with 72 for one of them and 67 for the other.**

2 **Can you find a way to arrange the nine digits so that they form a fraction equivalent to $\frac{1}{3}$?**

Flatback turtle facts



Connect the dots next to equivalent fractions. Each straight line drawn will pass through a letter and number, giving the puzzle answer code.



1	2	3	4	5	3	3	6	7	2	8	9	7	5	2	7	10	3	11	1
10	8	12	7	1	1	13	14	11	15	8	5	16	17	7	13	3	18	18	14	18	5	12
3	11	13	8	11	19	12	15	7	8	16	17	7	13	3	18	18	5	17	7
3	20	5	7	2	15	8	2	2	14	7	2	2	7	7	18	8	11	19
5	8	21	7	13	7	6	7	11	4	7	7	21	13	5	3	17	8	5	16	17

Simplifying fractions

Usually the best way to write fractions is in the simplest form. In other words, we reduce the fraction to its lowest equivalent form. To do this, it is necessary to divide both the numerator and the denominator by the same number.

The best number to divide by is the highest common factor of both the numerator and the denominator. The highest common factor (HCF) of two or more numbers is the largest factor that is the same for the given numbers. For example, the highest common factor of 8 and 12 is 4. This is because:

(a) 8 has the factors 1, 2, 4 and 8

(b) 12 has the factors 1, 2, 3, 4, 6 and 12.

The factors that are common to both 8 and 12 are 1, 2 and 4. The highest of these is 4. This means that 4 is the highest number that divides evenly into 8 and 12 without a remainder.

Dividing both the numerator and denominator by the same number is called *cancelling*.

WORKED Example 7

Write $\frac{3}{6}$ in simplest form.

THINK	WRITE
1 Write the fraction and find the highest common factor (HCF) or the largest number that is a factor of both the numerator and the denominator (3).	$\frac{3}{6}$
2 Cancel by dividing the numerator and denominator by this factor ($3 \div 3 = 1$, $6 \div 3 = 2$).	$= \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$
3 Write the answer in simplest form.	$= \frac{1}{2}$

If the largest factor is not used the first time, the process can be repeated to find the simplest fraction. To simplify a mixed numeral, leave the whole number and simplify the fraction.

WORKED Example 8

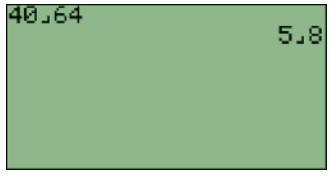
Simplify $7\frac{40}{64}$.

THINK	WRITE
1 Write the mixed numeral and think of the HCF or largest number that divides into both numerator and denominator; that is, the number 8.	$7\frac{40}{64}$
2 Divide both numerator and denominator by that factor ($40 \div 8 = 5$, $64 \div 8 = 8$).	$= 7\frac{\cancel{40}^{5}}{\cancel{64}_{8}}$
3 Write your answer as a mixed numeral in simplest form.	$= 7\frac{5}{8}$

Graphics Calculator tip! Simplifying fractions

Simplifying fractions

To reduce a fraction into its simplest form using a graphics calculator, first select RUN from the MENU. Type in the fraction using the $a\frac{b}{c}$ key and press EXE to obtain the answer. For the fraction $\frac{40}{64}$ in worked example 8, you would press the following keys:
4 0 $a\frac{b}{c}$ 6 4 EXE.



WORKED Example 9

Dipper's team scored 24 goals. Dipper scored 18 of the team's goals. What fraction of the team's goals did Dipper score? Simplify the answer.

THINK	WRITE
1 Write a fraction with Dipper's goals in the numerator and the team's scores in the denominator.	$\frac{18}{24}$
2 Simplify the fraction by dividing top and bottom by the HCF (6).	$= \frac{3}{4}$
3 Write the answer in a sentence.	Dipper scored $\frac{3}{4}$ of the team's goals.

remember

1. To simplify a fraction, divide both the numerator and denominator by the highest common factor or HCF.
2. To simplify a mixed numeral, leave the whole number and simplify the fraction.

EXERCISE 5B Simplifying fractions

SkillSheet 5.6

WORKED Example 7

1 Write the following fractions in simplest form.

a $\frac{5}{10}$ b $\frac{8}{12}$ c $\frac{21}{24}$ d $\frac{48}{60}$ e $\frac{28}{35}$

f $\frac{18}{24}$ g $\frac{81}{90}$ h $\frac{49}{56}$ i $\frac{100}{120}$ j $\frac{48}{50}$

k $\frac{63}{72}$ l $\frac{49}{70}$ m $\frac{33}{36}$ n $\frac{22}{50}$ o $\frac{21}{56}$

Excel Spreadsheet Simplifying a fraction

WORKED Example 8

2 Simplify.

a $4\frac{21}{35}$ b $7\frac{45}{54}$ c $10\frac{10}{20}$ d $5\frac{7}{42}$ e $3\frac{56}{64}$ f $1\frac{44}{48}$

Mathcad Simplifying a fraction

3 **multiple choice**

The fraction $\frac{26}{39}$ is the same as:

A $\frac{2}{3}$ B $\frac{2}{6}$ C $\frac{3}{8}$ D $\frac{9}{12}$

WORKED Example 9

4 Kylie's netball team scored 28 goals. Kylie scored 21 of her team's goals. What fraction of the team's goals did Kylie score? Simplify the answer.

5 Year 7 students at Springfield High School ran a car wash to raise money for the local hospital. They raised a total of $1000 and drew up a table showing how much money each class raised.

7A	$200	7B	$150	7C	$320
7D	$80	7E	$250		

Express as a simple fraction how much of the total each class raised.

6 Below are the results of a pie-eating competition.

Mark	5 pies
David	10 pies
Samantha	8 pies
Jules	6 pies
Ahmed	4 pies
Darren	7 pies

a How many pies were eaten in total?

b For each contestant, record the number of pies eaten as a fraction of the total number of pies eaten. Where possible, reduce the fractions to their simplest forms.

Improper fractions and mixed numerals

An *improper fraction* has a numerator that is larger than the denominator; for example, $\frac{3}{2}$. A mixed numeral is made up of a whole number and a fraction; for example, $1\frac{1}{2}$, $2\frac{3}{4}$.

Changing improper fractions to mixed numerals

Improper fractions can be changed to mixed numerals by making part of the fraction a whole number. Antonio Capricciosa's pizzas can show how it is done.

WORKED Example 10

Draw a diagram to show $\frac{5}{4}$ as pieces of pizza, then write the improper fraction as a mixed numeral.

THINK

1. Draw a whole pizza and divide it into the number of parts shown by the denominator.

WRITE

This is 4 quarters or $\frac{4}{4}$.

Continued over page

THINK	WRITE
2 Find the number of parts left over and draw them.	 Extra $\frac{1}{4}$ needed.
3 Write the improper fraction as a mixed numeral.	$\frac{5}{4} = 1\frac{1}{4}$

If the improper fraction had a larger numerator it might be necessary to draw more pizzas and the whole number part would then be greater than 1.

WORKED Example 11

Express $\frac{11}{5}$ as a mixed numeral.

THINK	WRITE
1 Write the improper fraction.	$\frac{11}{5}$
2 Find the largest multiple of the denominator which is less than the numerator (10) and write the improper fraction as two separate fractions.	$= \frac{10}{5} + \frac{1}{5}$
3 Simplify the new improper fraction and write the answer as a mixed numeral.	$= 2 + \frac{1}{5}$ $= 2\frac{1}{5}$

Changing mixed numerals to improper fractions

Mixed numerals can be changed to improper fractions by first changing the whole number to a fraction and then adding the two fractions. Antonio Capricciosa's pizzas can again show how it is done.

WORKED Example 12

Draw a diagram to show $2\frac{1}{3}$ as pieces of pizza and then write $2\frac{1}{3}$ as an improper fraction.

THINK	WRITE
1 Draw two whole pizzas and $\frac{1}{3}$ of a pizza. 2 Divide the whole pizzas into thirds.	
3 Count the number of thirds and write the mixed numeral as an improper fraction.	$2\frac{1}{3} = \frac{7}{3}$

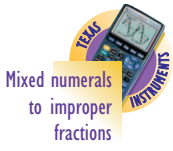
WORKED Example 13

Express $2\frac{3}{4}$ as an improper fraction.

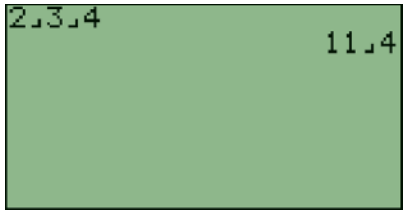
THINK	WRITE
1 Write the mixed numeral.	$2\frac{3}{4}$
2 Split the mixed numeral into the addition of a whole number and a fraction.	$= 2 + \frac{3}{4}$
3 Change the whole number to a fraction with the same denominator as the fraction part (4).	$= \frac{8}{4} + \frac{3}{4}$
4 Add the numerators of the two fractions to make one numerator and keep the denominator the same. The answer should be an improper fraction.	$= \frac{11}{4}$

Graphics Calculator tip! Changing a mixed numeral to an improper fraction

To change a mixed numeral to an improper fraction, enter the number using the fraction key. Then press SHIFT [d/c] to convert the mixed numeral into an improper fraction. For example, to change $2\frac{3}{4}$ to an improper fraction, select RUN from the MENU, press 2 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 4 and press EXE then SHIFT [d/c].



Mixed numerals to improper fractions



remember

1. An *improper fraction* has a numerator that is larger than the denominator.
2. A *mixed numeral* is made up of a whole number and a fraction.
3. Improper fractions can be changed to mixed numerals by making part of the fraction a whole number.
4. Mixed numerals can be changed to improper fractions by first changing the whole number to a fraction and then adding the two fractions.

EXERCISE 5C Improper fractions and mixed numerals

SKILLSHEET 5.7

WORKED Example 10

1 Draw a diagram to show the following improper fractions as pieces of pizza, then write each improper fraction as a mixed numeral.

a $\frac{5}{2}$ **b** $\frac{4}{3}$ **c** $\frac{11}{6}$

d $\frac{13}{8}$ **e** $\frac{12}{4}$ **f** $\frac{7}{6}$

2 Copy these statements, filling the gaps with the appropriate expression from the list below.

mixed numeral proper fraction improper fraction

a An _____ has a numerator that is larger than the denominator.

b A _____ consists of a whole number and a fraction.

c A _____ has a denominator that is larger than the numerator.

3 **a** Sarah ate $\frac{3}{2}$ or 3 halves of an apple. Draw the amount of apple that Sarah ate and express $\frac{3}{2}$ as a mixed numeral.

b Dean ate $\frac{9}{4}$ or 9 quarters of pizza. Draw the amount of pizza that Dean ate and express $\frac{9}{4}$ as a mixed numeral.

WORKED Example 11

EXCEL Spreadsheet Converting improper fractions to mixed numerals

Mathcad Converting improper fractions to mixed numerals

EXCEL Spreadsheet Converting improper fractions to mixed numerals (DIY)

4 Express these improper fractions as mixed numerals.

a $\frac{7}{5}$ **b** $\frac{11}{4}$ **c** $\frac{21}{2}$ **d** $\frac{39}{10}$

e $\frac{51}{12}$ **f** $\frac{92}{11}$ **g** $\frac{29}{13}$ **h** $\frac{23}{8}$

i $\frac{100}{3}$ **j** $\frac{25}{2}$ **k** $\frac{20}{3}$ **l** $\frac{10}{3}$

There is a faster way of changing improper fractions to mixed numerals. If the denominator is divided into the numerator, the answer is a whole number with a remainder. The whole number is part of the answer and the remainder becomes the numerator of the fractional part with the same denominator as the original improper fraction. For example, $\frac{5}{4} = 5 \div 4$

$= 1$ remainder 1

$= 1\frac{1}{4}$.

5 Change these improper fractions to mixed numerals.

a $\frac{8}{5}$ **b** $\frac{27}{8}$ **c** $\frac{11}{2}$ **d** $\frac{97}{9}$ **e** $\frac{58}{7}$

f $\frac{117}{10}$ **g** $\frac{67}{8}$ **h** $\frac{74}{9}$ **i** $\frac{31}{12}$ **j** $\frac{89}{6}$

6 **multiple choice**

a Which of the following is the same as $\frac{61}{8}$?

A $5\frac{5}{8}$ **B** $6\frac{5}{8}$ **C** $7\frac{5}{8}$ **D** $8\frac{5}{8}$

b Which of the following is the same as $\frac{73}{10}$?

A $3\frac{7}{10}$ **B** $7\frac{3}{10}$ **C** $7\frac{7}{10}$ **D** $3\frac{3}{10}$

c Which of the following is the same as $\frac{48}{5}$?

A $8\frac{4}{5}$ **B** $5\frac{5}{9}$ **C** $9\frac{5}{9}$ **D** $9\frac{3}{5}$

7 Kim and Carly arrived home from school quite hungry and cut up some fruit. Kim ate 9 quarters of an apple and Carly ate 11 quarters. How many apples did they each eat?

8 Daniel was selling pieces of quiche at the school fete. Each quiche was divided into 8 pieces. If Daniel sold 51 pieces, how many quiches did Daniel sell?

9 It was Max's responsibility to supply oranges for his football team at half-time. If 19 players ate $\frac{1}{4}$ orange each, how many oranges were eaten?

WORKED Example 12

10 Draw a diagram to show the following mixed numerals as pieces of pizza and then write each one as an improper fraction.

a $1\frac{1}{8}$ **b** $3\frac{3}{4}$ **c** $2\frac{1}{6}$

d $1\frac{1}{6}$ **e** $5\frac{1}{4}$ **f** $1\frac{3}{5}$

SkillSHEET 5.8

WORKED Example 13

11 Express the following mixed numerals as improper fractions.

a $1\frac{1}{7}$ **b** $1\frac{4}{5}$ **c** $2\frac{2}{3}$ **d** $3\frac{1}{3}$ **e** $6\frac{1}{4}$

f $4\frac{1}{9}$ **g** $11\frac{1}{2}$ **h** $3\frac{7}{8}$ **i** $7\frac{4}{5}$ **j** $9\frac{2}{5}$

EXCEL Spreadsheet
Converting mixed numerals to improper fractions

There is a quicker way of changing mixed numerals to improper fractions. Mixed numerals can be changed to improper fractions by multiplying the whole number and the denominator, then adding the numerator. The result becomes the numerator and the denominator stays the same.

Mathcad
Converting mixed numerals to improper fractions

For example, $4\frac{2}{3} = \frac{3 \times 4 + 2}{3}$

$= \frac{14}{3}$

12 Change these mixed numerals to improper fractions using the method above.

a $3\frac{1}{2}$ **b** $4\frac{1}{5}$ **c** $5\frac{4}{5}$ **d** $6\frac{5}{7}$ **e** $2\frac{9}{10}$

f $3\frac{7}{12}$ **g** $5\frac{3}{5}$ **h** $9\frac{4}{7}$ **i** $1\frac{10}{11}$ **j** $8\frac{5}{6}$

EXCEL Spreadsheet
Converting mixed numerals to improper fractions (DIY)

13 **multiple choice**

a Which of the following is the same as $10\frac{2}{7}$?

A $\frac{27}{10}$ **B** $\frac{72}{10}$ **C** $\frac{10}{27}$ **D** $\frac{72}{7}$

b Which of the following is the same as $8\frac{5}{6}$?

A $\frac{85}{6}$ **B** $\frac{58}{6}$ **C** $\frac{56}{8}$ **D** $\frac{53}{6}$

c Which of the following is the same as $7\frac{9}{11}$?

A $\frac{86}{11}$ **B** $\frac{99}{7}$ **C** $\frac{79}{11}$ **D** $\frac{97}{11}$

14 **a** Amy made apple pies for her Grandma's 80th birthday party. She divided each pie into 6 pieces, and after the party she noted that $4\frac{1}{6}$ pies had been eaten. How many pieces had been eaten?

b Amy's cousin Ria provided cordial for the same party, and calculated that she could make 20 drinks from each bottle. At the end of the party $3\frac{17}{20}$ bottles had been used. How many drinks had been consumed?

15 Insert the appropriate > or < sign between each pair of fractions to make a true statement.

a $\frac{8}{5}$ $1\frac{2}{5}$ **b** $\frac{19}{2}$ $8\frac{1}{2}$ **c** $\frac{33}{32}$ $1\frac{3}{32}$

d $\frac{15}{2}$ $5\frac{1}{2}$ **e** $3\frac{3}{4}$ $\frac{19}{4}$ **f** $7\frac{1}{7}$ $\frac{65}{7}$

g $2\frac{1}{3}$ $\frac{8}{3}$ **h** $1\frac{16}{17}$ $\frac{35}{17}$

10 QUICK QUESTIONS 1

1 Write down the equivalent fractions from this list:

$\frac{1}{4}$, $\frac{2}{6}$, $\frac{3}{12}$, $\frac{6}{25}$, $\frac{8}{32}$, $\frac{10}{41}$, $\frac{16}{64}$, $\frac{19}{76}$, $\frac{31}{124}$, $\frac{123}{490}$.

2 Write three equivalent fractions for $\frac{2}{5}$.

3 Write $\frac{12}{20}$ in simplest form.

4 On a recent quiz, Simone answered 6 out of the 50 questions incorrectly. What fraction of the test did Simone answer correctly? Express your answer in simplest form.

5 Write $\frac{26}{3}$ as a mixed numeral.

6 Change $1\frac{1}{4}$ to an improper fraction.

7 Write $\frac{12}{9}$ as a mixed numeral in simplest form.

8 Change $3\frac{6}{7}$ to an improper fraction.

9 Rewrite these fractions in ascending (from smallest to largest) order:

$\frac{2}{5}$, $\frac{14}{5}$, $1\frac{3}{5}$, $3\frac{3}{5}$, $\frac{15}{5}$, $4\frac{2}{5}$, $\frac{28}{5}$, $6\frac{1}{5}$.

10 A catering company provided 12 quiches which were each cut into 8 pieces for a luncheon. At the end of the function there were 12 pieces of quiche left over. How many whole quiches were eaten?

Adding and subtracting fractions

Fractions can be added and subtracted if they have the same denominator. Remember to simplify your answer if possible.

WORKED Example 14

Find $\frac{5}{9} + \frac{1}{9}$.

THINK	WRITE
1 Write the question.	$\frac{5}{9} + \frac{1}{9}$
2 If the denominators are the same, add or subtract the numerators. (Add.)	$= \frac{6}{9}$
3 Simplify the answer if possible.	$= \frac{2}{3}$

If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.

WORKED Example 15

Find $\frac{5}{6} - \frac{1}{12}$, expressing the answer in simplest form.

THINK	WRITE
1 Find multiples of each denominator.	Multiples of 6 are 6, 12 . . . Multiples of 12 are 12 . . .
2 Find the lowest common multiple (LCM) of 6 and 12 (12).	LCM is 12.
3 Rewrite the fractions if necessary so that they both have the LCM as the new denominator.	$\frac{5}{6} - \frac{1}{12}$ $= \frac{5 \times 2}{6 \times 2} - \frac{1}{12}$ $= \frac{10}{12} - \frac{1}{12}$
4 Subtract the numerators.	$= \frac{9}{12}$
5 Simplify your answer if necessary by cancelling by the highest common factor ($9 \div 3 = 3$, $12 \div 3 = 4$).	$= \frac{\cancel{9}^{3}}{\cancel{12}_{4}}$ $= \frac{3}{4}$

With practice many of these steps can be done in your head and the solution written as follows.

$$\frac{5}{6} - \frac{1}{12} = \frac{10}{12} - \frac{1}{12}$$
$$= \frac{9}{12}$$
$$= \frac{3}{4}$$

Adding and subtracting mixed numerals

There are a number of ways of adding and subtracting mixed numerals. Use the method which you prefer. All mixed numerals can be added and subtracted by first changing them to improper fractions. Remember to simplify the answer if possible.

WORKED Example 16

Find $2\frac{3}{4} + 1\frac{1}{2}$.

THINK	WRITE
1 Write the question.	$2\frac{3}{4} + 1\frac{1}{2}$
2 Change to improper fractions.	$= \frac{11}{4} + \frac{3}{2}$
3 Write both fractions with the same denominator using equivalent fractions.	$= \frac{11}{4} + \frac{6}{4}$
4 Add the fractions.	$= \frac{17}{4}$
5 Write the answer as a mixed numeral if appropriate.	$= 4\frac{1}{4}$

Alternatively, you can add the whole numbers first and then work out the fractional parts.

So, $2\frac{3}{4} + 1\frac{1}{2} = (2 + 1) + (\frac{3}{4} + \frac{1}{2})$

$= 3 + (\frac{3}{4} + \frac{2}{4})$

$= 3 + \frac{5}{4}$

$= 3 + 1\frac{1}{4}$

$= 4\frac{1}{4}$

This alternative method also works for subtracting fractions, unless the second fraction part is bigger than the first fraction part. Then use the method of equal addition. (See worked example 17.)

WORKED Example 17

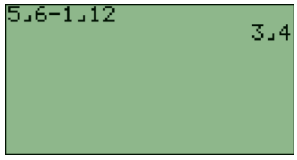
Find $3\frac{1}{5} - 1\frac{3}{4}$.

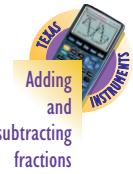
THINK	WRITE
1 Write the question.	$3\frac{1}{5} - 1\frac{3}{4}$
2 Write both fraction parts with the same denominator (20).	$= 3\frac{4}{20} - 1\frac{15}{20}$
3 Add the same number of parts to both fractions to make the second fraction a whole number $\left(\frac{5}{20}\right)$.	$= 3\frac{9}{20} - 2$
4 Subtract and simplify if necessary.	$= 1\frac{9}{20}$

Graphics Calculator tip!

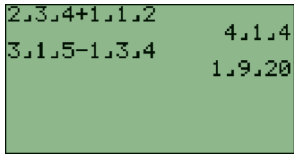
Adding and subtracting fractions

The calculation for worked example 15 would be entered as (5) ($a\frac{b}{c}$) (6) (−) (1) ($a\frac{b}{c}$) (1) (2) then press (EXE). This would appear on the screen as shown at right.





The calculations involving mixed numerals in worked examples 16 and 17 can be viewed in the screen shown at right. Note that the answers are shown as mixed numerals.



Estimation can be used to find approximate answers to addition and subtraction of mixed numerals. This allows you to easily check whether your answer to a calculation looks reasonable.

WORKED Example 18

Find $3\frac{1}{3} + 2\frac{1}{2}$ and check the answer by estimation.

THINK	WRITE
1 Write the question.	$3\frac{1}{3} + 2\frac{1}{2}$
2 Group the whole numbers and group the fractions.	$= (3 + 2) + (\frac{1}{3} + \frac{1}{2})$
3 Add the whole numbers and write the fractions with the same denominator.	$= 5 + (\frac{2}{6} + \frac{3}{6})$
4 Simplify the answer.	$= 5 + \frac{5}{6}$ $= 5\frac{5}{6}$
5 Check the answer by adding the whole numbers to find an approximation.	$3\frac{1}{3} + 2\frac{1}{2} > 3 + 2$ so $3\frac{1}{3} + 2\frac{1}{2} > 5$

remember

1. Fractions can be added and subtracted if they have the same denominator.
2. If fractions do not have the same denominator, use equivalent fractions to make the denominators the same.
3. Mixed numerals can be added or subtracted by first changing them to improper fractions.
4. Mixed numerals can also be added or subtracted by working with the whole numbers, then the fractional parts. If the second fractional part is bigger than the first fractional part, make the second fraction equal to a whole number by adding the same fraction to both parts.

EXERCISE 5D Adding and subtracting fractions

SkillSheet 5.9

WORKED Example 14

1 Find:

a $\frac{1}{5} + \frac{3}{5}$ **b** $\frac{9}{11} - \frac{3}{11}$ **c** $\frac{3}{6} + \frac{2}{6}$

d $\frac{20}{50} + \frac{11}{50}$ **e** $\frac{15}{25} - \frac{6}{25}$ **f** $\frac{12}{12} - \frac{1}{12}$

g $\frac{5}{8} - \frac{1}{8} + \frac{3}{8}$ **h** $\frac{4}{7} + \frac{5}{7} - \frac{3}{7}$ **i** $\frac{33}{100} + \frac{7}{100} - \frac{11}{100}$

EXCEL Spreadsheet: Adding and subtracting fractions

2 Find the following, expressing the answers in simplest form.

a $\frac{5}{12} + \frac{1}{12}$ **b** $\frac{7}{16} - \frac{3}{16}$ **c** $\frac{9}{15} - \frac{4}{15}$

d $\frac{37}{100} + \frac{13}{100}$ **e** $\frac{9}{4} - \frac{7}{4}$ **f** $\frac{4}{7} + \frac{5}{7} - \frac{2}{7}$

g $\frac{13}{16} + \frac{9}{16} - \frac{10}{16}$ **h** $\frac{11}{28} + \frac{10}{28}$ **i** $\frac{21}{81} + \frac{21}{81} + \frac{30}{81}$

Mathcad: Adding fractions

Mathcad: Subtracting fractions

WORKED Example 15

3 Find the following, expressing the answers in simplest form.

a $\frac{1}{2} + \frac{1}{4}$ **b** $\frac{1}{2} - \frac{1}{4}$ **c** $\frac{1}{3} + \frac{2}{21}$

d $\frac{1}{8} + \frac{20}{32}$ **e** $\frac{9}{18} + \frac{1}{6}$ **f** $\frac{17}{40} + \frac{1}{2}$

g $\frac{7}{9} - \frac{8}{27}$ **h** $\frac{31}{35} - \frac{3}{7}$ **i** $\frac{8}{10} - \frac{28}{70}$

SkillSheet 5.4

4 Find the lowest common multiple of each of the following pairs of numbers.

a 2 and 3 **b** 3 and 5 **c** 5 and 10

d 4 and 6 **e** 5 and 6 **f** 9 and 6

g 4 and 8 **h** 6 and 8 **i** 7 and 5

5 Using the lowest common multiples that were found in question **4** as denominators, add or subtract these fractions.

a $\frac{1}{2} - \frac{1}{3}$ **b** $\frac{2}{3} - \frac{2}{5}$ **c** $\frac{3}{5} + \frac{3}{10}$

d $\frac{3}{4} - \frac{1}{6}$ **e** $\frac{4}{5} - \frac{1}{6}$ **f** $\frac{1}{9} + \frac{5}{6}$

g $\frac{1}{4} + \frac{3}{8}$ **h** $\frac{5}{6} - \frac{3}{8}$ **i** $\frac{5}{7} - \frac{2}{5}$

6 Answer the following questions by first finding the lowest common denominator. Simplify your answer if necessary.

a $\frac{2}{3} + \frac{1}{4}$ **b** $\frac{5}{7} - \frac{3}{8}$ **c** $\frac{4}{5} - \frac{3}{4}$

d $\frac{7}{10} + \frac{2}{15}$ **e** $\frac{5}{8} - \frac{5}{12}$ **f** $\frac{7}{9} - \frac{1}{2}$

g $\frac{5}{12} + \frac{2}{9}$ **h** $\frac{7}{11} + \frac{1}{4}$ **i** $\frac{4}{5} - \frac{2}{11}$

j $\frac{5}{6} + \frac{7}{10}$ **k** $\frac{3}{10} - \frac{1}{20}$ **l** $\frac{7}{8} - \frac{3}{5}$

m $\frac{2}{3} + \frac{4}{5}$ **n** $\frac{5}{7} + \frac{3}{4}$ **o** $\frac{5}{9} - \frac{2}{7}$

p $\frac{2}{3} - \frac{1}{13}$ **q** $\frac{11}{12} - \frac{7}{10}$ **r** $\frac{4}{7} + \frac{5}{6}$

7 Angela was a little hungry the day before her birthday, and spied the cake to be eaten the following day. She decided that if she ate only $\frac{1}{10}$ of it, her Mum might not notice. However, she was still hungry and took another $\frac{1}{6}$.

a How much of the cake did Angela eat before her birthday?

b How much of the cake was left for her birthday?

8 **multiple choice**

a The lowest common denominator of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ is:

A 16 **B** 24 **C** 18 **D** 12

b The lowest common denominator of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{5}$ is:

A 8 **B** 10 **C** 20 **D** 30

9 Answer the following:

a $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$ **b** $\frac{3}{10} + \frac{4}{15} - \frac{1}{6}$ **c** $\frac{7}{8} - \frac{3}{4} + \frac{4}{6}$

d $\frac{11}{12} - \frac{8}{15} - \frac{3}{20}$ **e** $\frac{1}{16} + \frac{1}{8} + \frac{1}{4}$ **f** $\frac{5}{6} - \frac{7}{18} - \frac{1}{9}$

g $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ **h** $\frac{3}{4} + \frac{1}{3} + \frac{5}{6}$

Mathcad: Adding and subtracting fractions

10 A cake needed to be cooked at 200°C for $\frac{3}{4}$ hour then a further $\frac{1}{2}$ hour at 150°C. What is the total cooking time in hours?

11 Robert, Jason and Luke play football in the same team. Last Saturday, Robert kicked $\frac{1}{4}$ of the team's goals, Jason kicked $\frac{1}{3}$ and Luke $\frac{5}{12}$. What fraction of the team's goals were kicked by:

a Jason and Luke? **b** Robert and Jason? **c** the three boys together?

12 Jacinta and her friend decided to climb the tree in her Nan's front yard. It took her 10 minutes to climb halfway up, 10 more minutes to climb another sixth of the way and yet another 10 minutes to reach a further tenth of the way. Here she sat admiring the view for 5 minutes before beginning her descent.

a What fraction of the height of the tree had Jacinta climbed after 20 minutes?

b What fraction of the tree did Jacinta climb in total?

WORKED Example 16

13 Find:

a $1\frac{1}{5} + 3\frac{1}{2}$ **b** $3\frac{2}{3} - 2\frac{1}{2}$ **c** $5\frac{2}{3} - 2\frac{1}{4}$

d $9\frac{3}{8} + 4$ **e** $2\frac{3}{5} - 1\frac{1}{2}$ **f** $3\frac{1}{2} + 2\frac{3}{4}$

WORKED Example 17

14 Find:

a $7 - 5\frac{2}{3}$ **b** $2\frac{1}{3} - \frac{5}{6}$ **c** $3\frac{1}{2} - 1\frac{7}{8}$ **d** $4\frac{3}{5} - 2\frac{9}{10}$ **e** $5\frac{1}{4} - 1\frac{7}{12}$ **f** $5\frac{3}{4} - 3\frac{5}{6}$

15 Find:

a $\frac{1}{5} + 2\frac{2}{7}$ **b** $\frac{5}{8} + 3\frac{3}{4} - \frac{1}{2}$ **c** $2\frac{2}{9} + 1\frac{1}{3} + \frac{1}{6}$ **d** $4\frac{2}{3} + 1\frac{3}{8} - 3$

WORKED Example 18

16 Find the following and check the answers by estimation.

a $5\frac{1}{3} + 2\frac{1}{4}$ **b** $1\frac{1}{6} + 2\frac{1}{2}$ **c** $3\frac{1}{3} + 1\frac{1}{6}$ **d** $6\frac{1}{2} - 2\frac{1}{4}$ **e** $4\frac{3}{5} - 1\frac{3}{10}$ **f** $7\frac{5}{6} - 3\frac{1}{3}$

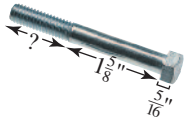
17 One-third of a litre of cordial is mixed with $1\frac{1}{2}$ litres of water. How many litres of drink have been made?

18 At a school dance-a-thon, the number of hours each student danced was recorded.

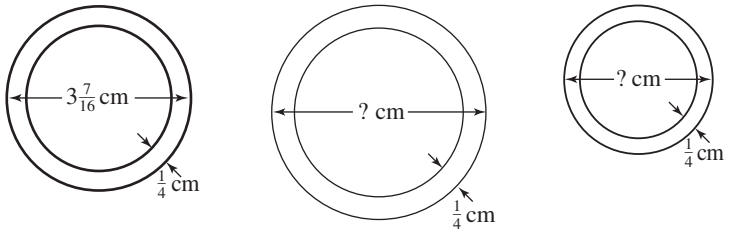
Matthew $1\frac{3}{4}$ hours, Leigh $2\frac{2}{3}$ hours, Daniel $1\frac{5}{6}$ hours

How many hours did the boys dance in total?

19 The lengths of bolts and nails are often measured in inches. For the bolt shown, find the length of the threaded section if the total length of the bolt is $3\frac{1}{4}$ inches.



20 To make the casing for a simple collapsible telescope you need two cylinders, one of which fits inside the other. One cylinder has measurements as shown below left.



a What is the width of a cylinder (see middle diagram above) which would slide over the one shown above left?

b What is the width of a cylinder (see diagram above right) which would slide into the one shown above left?

1 Find the fraction that is equivalent to $\frac{3}{5}$ with the product of its numerator and denominator being equal to 375.

2 The sum of two fractions is $1\frac{1}{2}$. Their difference is $\frac{5}{12}$. What are the fractions?

3 If a clock takes 5 seconds to strike 6 o'clock, how many seconds will it take to strike 12 o'clock?

4 Jeff wrote $\frac{3}{4} + \frac{1}{5} = \frac{4}{9}$. Is he correct? Explain your answer.

Egyptian fractions

As we saw in chapter 1 (pages 3–4), the ancient Egyptians (from 3100 BC to 332 BC) used different symbols or hieroglyphs to show numbers.

Using these symbols, the number 23 462 would be shown as:


23 462

1 Work out each number using the description given above.

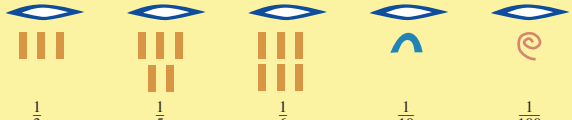
a 

b 

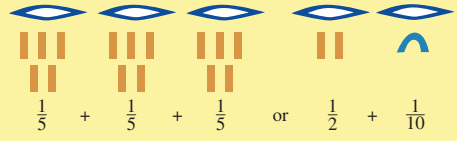
c 

d 

As well as expressing words and whole numbers with hieroglyphics, the ancient Egyptians devised a way of expressing fractions. They placed a 'mouth' (pronounced 'er') over their symbols for whole numbers. For example:



Apart from some special fractions, all numerators were of the value 1, so they would express a fraction like $\frac{3}{5}$ as shown at right.



2 What do these symbols represent?

a 

b 

c 

d 

3 Perform the following calculations. Give your answer in Egyptian hieroglyphics and in Hindu–Arabic numerals.

a Add these two numbers.

b Subtract the second number from the first.

4 Write four of your own questions using Egyptian hieroglyphics. Swap these with a partner. Calculate the answers to your partner's set of questions, showing clearly how you were able to work them out.

Multiplying fractions

To multiply fractions, multiply the numerators and multiply the denominators.

WORKED Example 19

Find $\frac{1}{5} \times \frac{3}{4}$.

THINK	WRITE
1 Write the question.	$\frac{1}{5} \times \frac{3}{4}$
2 Write as one fraction.	$= \frac{1 \times 3}{5 \times 4}$
3 Multiply the numerators and the denominators and simplify your answer if appropriate.	$= \frac{3}{20}$

Fractions can be simplified by dividing both numerator and denominator by the same number before being multiplied. This is easier than working with large numbers.

Remember that dividing both numerator and denominator by the same number is often called *cancelling*.

WORKED Example 20

Find $\frac{2}{3} \times \frac{9}{10}$.

THINK	WRITE
1 Write the question.	$\frac{2}{3} \times \frac{9}{10}$
2 Write the two fractions as one keeping the multiplication signs.	$= \frac{{}^{1}\cancel{2} \times \cancel{9}^{3}}{\cancel{3}_{1} \times \cancel{10}^{5}}$
3 Divide the numerator and denominator by the same numbers or cancel.	
4 Multiply to find the answer.	$= \frac{3}{5}$

WORKED Example 21

Find $\frac{1}{3} \times 36$.

THINK	WRITE
1 Write the question.	$\frac{1}{3} \times 36$
2 Express the whole number as a fraction and cancel if possible.	$= \frac{1}{\cancel{3}_{1}} \times \frac{\cancel{36}^{12}}{1}$
3 Multiply the numerators and denominators.	$= \frac{12}{1}$
4 Simplify your answer.	$= 12$

Mixed numerals must be changed to improper fractions before multiplying.

WORKED Example 22

Find $1\frac{1}{2} \times 3\frac{3}{5}$.

THINK	WRITE
1 Write the question.	$1\frac{1}{2} \times 3\frac{3}{5}$
2 Change mixed numerals to improper fractions and cancel if possible.	$= \frac{3}{\cancel{2}_{1}} \times \frac{\cancel{18}^{9}}{5}$
3 Multiply numerators and denominators.	$= \frac{27}{5}$
4 Simplify your answer.	$= 5\frac{2}{5}$

Using the word 'of'

Sometimes the word *of* can mean the same as *multiply*. Antonio might take $\frac{1}{3}$ of an hour (60 minutes) to cook his pizzas. A third of an hour, or $\frac{1}{3}$ of 60 minutes, is the same as $\frac{1}{3} \times \frac{60}{1} = 20$ minutes. So it takes Antonio 20 minutes to cook a pizza.

A customer wants to buy $\frac{1}{4}$ of the 8 remaining pizzas for the family's evening meal. A quarter of 8 is the same as $\frac{1}{4} \times \frac{8}{1} = 2$. So the customer buys 2 pizzas.

WORKED Example 23

Find $\frac{2}{3}$ of $\frac{1}{4}$.

THINK	WRITE
1 Write the question.	$\frac{2}{3}$ of $\frac{1}{4}$
2 Change 'of' to '×' and cancel if appropriate.	$= \frac{\cancel{2}^{1}}{3} \times \frac{1}{\cancel{4}_{2}}$
3 Perform the multiplication.	$= \frac{1}{6}$

WORKED Example 24

Mum put a 2-litre carton of flavoured milk in the refrigerator. When the children came home from school, Joanna drank one quarter of the milk, Kieren drank one half of it and Daisy drank one sixth of it. What fraction of the milk, in litres, did each person drink?

THINK	WRITE
1 Write the fraction of the milk that Joanna drank and simplify the answer.	Joanna: $\frac{1}{4}$ of $2 = \frac{1}{4} \times 2$ $= \frac{1}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{1}$ $= \frac{1}{2}$
2 Write the fraction of the milk that Kieren drank and simplify the answer.	Kieren: $\frac{1}{2}$ of $2 = \frac{1}{2} \times 2$ $= \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{2}^{1}}{1}$ $= 1$
3 Write the fraction of the milk that Daisy drank and simplify the answer.	Daisy: $\frac{1}{6}$ of $2 = \frac{1}{6} \times 2$ $= \frac{1}{\cancel{6}_{3}} \times \frac{\cancel{2}^{1}}{1}$ $= \frac{1}{3}$
4 Write a sentence giving the answer in litres.	Joanna drank a half a litre, Kieren drank 1 litre and Daisy drank a third of a litre.

remember

1. To multiply fractions, multiply the numerators and multiply the denominators.
2. Fractions can be simplified (or cancelled) before being multiplied.
3. A whole number can be written as a fraction with a denominator of 1.
4. Change mixed numerals to improper fractions before multiplying.
5. In a problem involving fractions, 'of' can be replaced with 'multiply', then the fractions can be multiplied in the usual way.

EXERCISE 5E Multiplying fractions

Mathcad: Multiplying fractions

EXCEL Spreadsheet: Multiplying fractions

WORKED Example 19

1 Find:

a $\frac{1}{2} \times \frac{1}{4}$ b $\frac{1}{3} \times \frac{2}{3}$ c $\frac{2}{5} \times \frac{3}{5}$ d $\frac{5}{9} \times \frac{4}{3}$

e $\frac{11}{13} \times \frac{1}{2}$ f $\frac{6}{7} \times \frac{9}{11}$ g $\frac{11}{12} \times \frac{11}{12}$ h $\frac{8}{9} \times \frac{5}{9}$

i $\frac{5}{8} \times \frac{1}{3}$ j $\frac{2}{3} \times \frac{2}{3}$ k $\frac{5}{6} \times \frac{11}{12}$ l $\frac{5}{11} \times \frac{5}{12}$

m $\frac{7}{8} \times \frac{7}{8}$ n $\frac{2}{3} \times \frac{4}{13}$ o $\frac{3}{5} \times \frac{3}{10}$ p $\frac{4}{7} \times \frac{2}{3}$

WORKED Example 20

2 Find:

a $\frac{4}{8} \times \frac{3}{9}$ **b** $\frac{2}{5} \times \frac{1}{2}$ **c** $\frac{3}{7} \times \frac{5}{6}$

d $\frac{3}{8} \times \frac{4}{15}$ **e** $\frac{10}{11} \times \frac{22}{25}$ **f** $\frac{24}{27} \times \frac{9}{8}$

g $\frac{18}{32} \times \frac{64}{72}$ **h** $\frac{48}{56} \times \frac{24}{60}$ **i** $\frac{15}{27} \times \frac{36}{45}$

j $\frac{8}{49} \times \frac{14}{16}$ **k** $\frac{4}{9} \times \frac{6}{8}$ **l** $\frac{6}{12} \times \frac{8}{10}$

m $\frac{42}{54} \times \frac{12}{49}$ **n** $\frac{5}{25} \times \frac{12}{48}$ **o** $\frac{18}{24} \times \frac{12}{36}$

3 Mark's uncle gave him a family-size block of chocolate. He divided it into thirds to share with his two sisters. His friends Tom and Nick then arrived to visit, and Mark shared his portion of the chocolate with them.

a Draw the block of chocolate, and shade Mark's third of the block. Then draw lines to represent Tom's share.

b What fraction of the block of chocolate did Nick receive?

WORKED Example 21

4 Find:

a $\frac{2}{3} \times 3$ **b** $\frac{5}{14} \times 7$ **c** $\frac{15}{22} \times 11$ **d** $\frac{5}{16} \times 4$

WORKED Example 22

5 Find:

a $1\frac{1}{4} \times \frac{2}{3}$ **b** $\frac{3}{4} \times 1\frac{5}{8}$ **c** $1\frac{1}{2} \times 1\frac{1}{2}$ **d** $2\frac{5}{8} \times 1\frac{3}{4}$

e $2\frac{1}{5} \times 3\frac{1}{2}$ **f** $10\frac{9}{10} \times \frac{2}{3}$ **g** $2\frac{1}{4} \times 2\frac{1}{2}$ **h** $\frac{4}{5} \times 1\frac{6}{7}$

i $\frac{9}{10} \times 3\frac{2}{3}$ **j** $4\frac{1}{3} \times \frac{3}{5}$ **k** $1\frac{1}{2} \times 12$ **l** $2\frac{3}{4} \times 6$

m $5 \times 3\frac{1}{2}$ **n** $8 \times 3\frac{1}{4}$ **o** $4\frac{3}{5} \times 2\frac{1}{10}$ **p** $3\frac{5}{6} \times 9\frac{5}{7}$

q $5\frac{1}{5} \times 6\frac{2}{3}$ **r** $3\frac{3}{4} \times 2\frac{2}{3}$

6 Tim was looking for something to do one Saturday afternoon and his Dad suggested he cook something for afternoon tea.
Tim found a recipe for peanut butter muffins.
The recipe is given below.

Ingredients:

$\frac{1}{4}$ cup sugar, $\frac{1}{4}$ cup margarine,

$\frac{1}{2}$ cup peanut butter, 2 eggs,

$1\frac{1}{2}$ cups milk, $2\frac{1}{2}$ cups self-raising flour,

$\frac{1}{4}$ teaspoon baking soda

Method:

Blend the sugar, margarine and peanut butter. Beat in the eggs and milk. Add the self-raising flour and baking soda. Place the mixture in greased muffin pans and sprinkle with cinnamon sugar. Bake for 15–20 minutes in a 200°C oven.

This recipe makes 10 muffins, but Tim wants enough mixture to make 15. He needs to multiply the quantities by $1\frac{1}{2}$. Write the quantities Tim needs to make 15 muffins.

Mathcad — Fraction of a number

WORKED Example 23

7 Find:

a $\frac{1}{2}$ of $\frac{1}{4}$ **b** $\frac{3}{4}$ of $\frac{2}{3}$ **c** $\frac{5}{6}$ of $\frac{5}{6}$

d $\frac{9}{10}$ of $\frac{5}{7}$ **e** $\frac{8}{9}$ of $\frac{1}{4}$ **f** $\frac{5}{7}$ of $\frac{7}{25}$

g $\frac{4}{5}$ of 20 **h** $\frac{7}{8}$ of 32 **i** $\frac{9}{10}$ of 50

j $\frac{2}{5}$ of 45 **k** $\frac{3}{5}$ of 25 **l** $\frac{8}{9}$ of 81

m $\frac{3}{4}$ of 6 **n** $\frac{9}{10}$ of 15 **o** $\frac{1}{12}$ of 10

p $\frac{3}{7}$ of 12 **q** $\frac{1}{2}$ of $1\frac{1}{2}$ **r** $\frac{3}{4}$ of $8\frac{2}{3}$

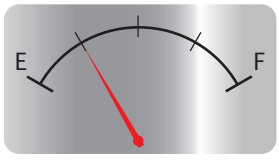
s $\frac{2}{3}$ of $\frac{3}{2}$ **t** $\frac{5}{6}$ of $\frac{18}{25}$ **u** $\frac{7}{8}$ of $3\frac{1}{3}$

GAME time — Fractions — 002

8 A car's petrol tank holds 48 litres of fuel.

a If the tank was full at the start of a trip, what fraction of the tank has been used?

b How many litres of petrol have been used?



WORKED Example 24

9 Jamie's Mum made 6 litres of punch for his birthday party. Emily drank $\frac{1}{3}$ of the punch, Tracy drank $\frac{1}{12}$ and Jonathan drank $\frac{1}{18}$. How much punch, in litres, did each person drink?

10 Zoe and Sarah play basketball with the Sharp Shooters. The games are played for 40 minutes. Zoe played $\frac{4}{5}$ of last week's game and Sarah played $\frac{7}{8}$. How many minutes of the game did:

a Zoe play?

b Sarah play?

11 Gustave's monthly take-home pay is $2400. From this he spends a quarter on his home loan payments, one half on food and drink and one sixth on clothing. One half of the remainder goes into his savings account. How much money does Gustave put into his savings account each month?

12 To make a fruit loaf using an automatic bread maker, the fruit needs to be added after the second rise stage and the top of the bread needs to be glazed two-thirds of the way through the bake stage. The instruction booklet shows you how long each stage of the total cycle takes.

Stage	Length of time
First knead	5 minutes
Second knead	20 minutes
First rise then punch down	40 minutes
Second rise then punch down	25 minutes
Third rise	50 minutes
Bake	40 minutes

a After what length of time should the fruit be added?

b Express this as a fraction of the total time in the bread maker.

c After what length of time should the top of the bread be glazed?

13 **a** Draw a quarter of a pie.
b Shade half of this piece of pie.
c What fraction of the whole pie is shaded?
d Complete the mathematical sentence $\frac{1}{4} \times \frac{1}{2} =$ __ to show how the answer was found.

14 **a** Find how many half pizzas in 1 pizza.
b Find how many half pizzas in 2 whole pizzas.
c Find how many half pizzas in 5 whole pizzas.

Musical fractions

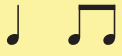
In music, particular symbols tells us how long a note is to be held for. The time value of each note is shown below.

4 beats (semibreve)	2 beats (minim)	1 beat (crotchet)	$\frac{1}{2}$ beat (quaver)	$\frac{1}{4}$ beat (semi-quaver)

More than 1 quaver is shown as 

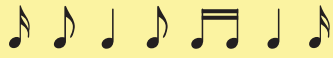
More than 1 semi-quaver is shown as 

1 What is the total number of beats indicated by these notes?

a 

b 

c 

d 

When musicians decide they want a note held for a little longer, they put a dot after the note. The dot adds an extra half of the number of beats to the original time value. For example, a dotted minim has a time value of $2 + \frac{1}{2} \times 2 = 3$ beats.

2 What is the time value indicated by a dotted crotchet?
3 What is the time value indicated by a dotted quaver?
4 What is the total number of beats indicated by these notes?

a 

b 

5 Write one note which is equivalent in time value to each of the following.

a 

b 

c 

d 

e 

f 

g 

h 

To make music more organised, time signatures were added to the beginning of a piece of music so that each section or bar was divided up into equal parts. For example, $\frac{4}{4}$ tells us that we need 4 crotchet beats (or notes that will make up that amount) in a bar. The 4 on the top shows us the number of beats per bar, while the 4 on the bottom shows us that the note values we use are the ones that belong to crotchet time. In $\frac{3}{4}$ time, we need 3 beats to make up a bar and in $\frac{2}{4}$ time we need 2 beats to make up a bar.

6 Add one note to complete each of the following bars.

a 2/4 

b 4/4 

c 3/4 

d 2/4 

e 4/4 

f 3/4 

g 3/4 

h 2/4 

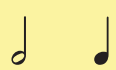
7 Add 2 notes to complete each of the following bars.

a 4/4 

b 3/4 

c 2/4 

d 3/4 

e 4/4 

f 2/4 

g 3/4 

h 3/4 

8 Put time signatures to each of the following rhythm patterns. (They are crotchet beats.)

a 

b 

9 Write your own rhythm patterns 4 bars long, using a different pattern in each bar for each of the following time signatures.

a 3/4 **b** 4/4

Dividing fractions

The *reciprocal* of a fraction is found by tipping the fraction upside down. So $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$.

A whole number can be written as a fraction by putting it over 1 so that $\frac{1}{4}$ is the reciprocal of $\frac{4}{1}$.

WORKED Example 25

Find the reciprocal of $\frac{2}{3}$.

THINK	WRITE
Turn the fraction upside down and write a sentence.	The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

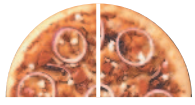
To find the reciprocal of a mixed numeral, express it as an improper fraction first.

WORKED Example 26

Find the reciprocal of $1\frac{2}{3}$.

THINK	WRITE
1 Write $1\frac{2}{3}$ as an improper fraction.	$1\frac{2}{3} = \frac{5}{3}$
2 Turn it upside down and write a sentence.	The reciprocal of $1\frac{2}{3}$ is $\frac{3}{5}$.

Antonio had a half of one of his famous pizzas left. A small child asked him how many quarters there were in his half. Antonio cut the pizza as shown, so that it was clear that one half divided into quarters gave two pieces or two quarters.



$$\frac{1}{2} \div \frac{1}{4} = 2$$

The answer could also have been found by changing the division sign to a multiplication sign and tipping the second fraction upside down.

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$$
$$= 2$$

Division by a number is the same as multiplication by its reciprocal.

WORKED Example 27

Find $\frac{2}{3} \div \frac{4}{9}$.

THINK	WRITE
1 Write the question.	$\frac{2}{3} \div \frac{4}{9}$
2 Change the division sign to a multiplication sign and tip the second fraction.	$= \frac{2}{3} \times \frac{9}{4}$
3 Cancel if possible.	$= \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{4}_{2}}$
4 Perform multiplication.	$= \frac{3}{2}$
5 Simplify your answer.	$= 1\frac{1}{2}$

To divide mixed numerals, you can first change them to improper fractions.

WORKED Example 28

Find $1\frac{1}{2} \div 3\frac{2}{5}$.

THINK	WRITE
1 Write the question.	$1\frac{1}{2} \div 3\frac{2}{5}$
2 Change mixed numerals to improper fractions.	$= \frac{3}{2} \div \frac{17}{5}$
3 Change the ÷ to × and tip the second fraction.	$= \frac{3}{2} \times \frac{5}{17}$
4 Multiply the numerators and the denominators and simplify your answer if necessary.	$= \frac{15}{34}$

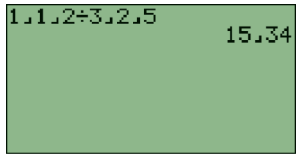
Graphics Calculator tip!

Dividing a fraction by a fraction

Dividing a fraction by a fraction

Dividing a fraction by a fraction can be done using the $a\frac{b}{c}$ key to enter the fractions and the ÷ key for the division.

The calculation for $1\frac{1}{2} \div 3\frac{2}{5}$ is shown.



remember

1. The *reciprocal* of a fraction is found by tipping the fraction upside down.
2. The reciprocal of a mixed numeral is found by expressing it as an improper fraction, then tipping it upside down.
3. A whole number can be written as a fraction by putting it over 1.
4. Division by a number is the same as multiplication by its reciprocal.
5. To divide mixed numerals, first change them to improper fractions.

EXERCISE 5F Dividing fractions

WORKED Example 25

Reciprocals

1 Find the reciprocals of each of the following.

a $\frac{3}{4}$	**b** $\frac{2}{7}$	**c** $\frac{5}{3}$	**d** $\frac{5}{12}$
e $\frac{6}{5}$	**f** $\frac{9}{2}$	**g** $\frac{5}{3}$	**h** $\frac{10}{3}$
i $\frac{1}{5}$	**j** $\frac{1}{12}$	**k** $\frac{3}{15}$	**l** $\frac{2}{22}$
m 5	**n** 20	**o** 13	**p** 1

WORKED Example 26

2 Find the reciprocals of these mixed numerals.

a $1\frac{2}{3}$ **b** $4\frac{3}{7}$ **c** $7\frac{1}{2}$ **d** $4\frac{2}{5}$ **e** $9\frac{2}{7}$

f $3\frac{6}{7}$ **g** $3\frac{3}{8}$ **h** $5\frac{1}{3}$ **i** $10\frac{2}{9}$ **j** $6\frac{9}{10}$

3 Multiply each of these numbers by its reciprocal.

a $\frac{5}{7}$ **b** $\frac{2}{9}$ **c** $\frac{3}{5}$ **d** $3\frac{2}{7}$ **e** $2\frac{1}{8}$ **f** $3\frac{3}{4}$

4 Copy and complete the following statement:

When any number is multiplied by its _____, the answer is always _____.

WORKED Example 27

5 Find:

a $\frac{1}{4} \div \frac{2}{3}$ **b** $\frac{8}{9} \div \frac{7}{6}$ **c** $\frac{9}{10} \div \frac{1}{2}$

d $\frac{1}{12} \div \frac{2}{3}$ **e** $\frac{4}{11} \div \frac{5}{6}$ **f** $\frac{7}{12} \div \frac{9}{11}$

g $\frac{10}{3} \div \frac{15}{21}$ **h** $\frac{2}{7} \div \frac{20}{21}$ **i** $\frac{8}{15} \div \frac{16}{15}$

j $\frac{4}{9} \div \frac{4}{9}$ **k** $\frac{4}{7} \div 10$ **l** $\frac{5}{8} \div 20$

m $\frac{4}{3} \div 12$ **n** $\frac{15}{11} \div 5$ **o** $\frac{7}{9} \div 30$

Mathcad Dividing fractions

EXCEL Spreadsheet Dividing fractions

WORKED Example 28

6 Find:

a $1\frac{1}{4} \div \frac{3}{4}$ **b** $9 \div 4\frac{1}{2}$ **c** $\frac{9}{4} \div 3\frac{1}{2}$

d $\frac{10}{3} \div 3\frac{1}{3}$ **e** $5 \div 2\frac{3}{4}$ **f** $7\frac{4}{9} \div \frac{1}{3}$

g $\frac{1}{8} \div 2\frac{1}{2}$ **h** $1\frac{1}{5} \div 2\frac{1}{3}$ **i** $5\frac{6}{7} \div 2\frac{3}{4}$

j $1\frac{1}{2} \div \frac{1}{2}$ **k** $10\frac{9}{10} \div 4$ **l** $4\frac{1}{2} \div 3\frac{6}{7}$

m $3\frac{2}{7} \div 2\frac{1}{7}$ **n** $4\frac{2}{3} \div \frac{7}{6}$ **o** $3\frac{5}{8} \div 1\frac{3}{4}$

7 Felicity is dividing $1\frac{1}{2}$ kilograms of mince steak into 8 hamburger portions. How many kilograms of mince will be in each hamburger portion?

8 Ned spends his school holidays helping his father with the shearing.

a It takes Ned $\frac{1}{4}$ hour to shear one sheep. How many sheep will Ned shear in $5\frac{1}{2}$ hours?

b Ned's father, Wesley, shears a sheep every $\frac{1}{12}$ of an hour. How many sheep would Wesley shear if he worked continuously for 8 hours?

9 Michelle can run one lap of a cross-country course in $\frac{1}{5}$ hour.

a How many laps could she complete in $1\frac{1}{3}$ hours? (Assume she can keep up the same pace.)

b How many minutes does it take Michelle to run one lap?

WORKSHEET 5.2

10 QUICK QUESTIONS 2

Calculate each of the following, expressing your answer in simplest form.

1 $\frac{12}{15} - \frac{4}{15}$ **2** $\frac{5}{6} + \frac{3}{10}$ **3** $3\frac{1}{4} + 2\frac{2}{5}$ **4** $11\frac{5}{8} - 3\frac{1}{2}$ **5** $\frac{12}{20} \times \frac{4}{18}$

6 $3\frac{5}{9} \times \frac{15}{20}$ **7** $\frac{2}{7}$ of $\frac{21}{40}$ **8** $\frac{5}{9} \div \frac{2}{3}$ **9** $8 \div 1\frac{3}{4}$ **10** $4\frac{2}{5} \div \frac{2}{11}$

Mixed operations with fractions

If more than one operation is involved in a question with fractions, the same set order is used as with whole numbers. Therefore, the order of operations is:

1. grouping symbols
2. multiplication and division (from left to right)
3. addition and subtraction (from left to right).

WORKED Example 29

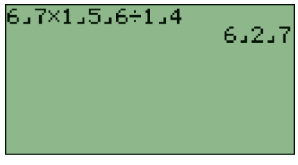
Find $\frac{6}{7} \times 1\frac{5}{6} \div \frac{1}{4}$.

THINK	WRITE
1 Write the question.	$\frac{6}{7} \times 1\frac{5}{6} \div \frac{1}{4}$
2 Change the mixed numeral to an improper fraction.	$= \frac{6}{7} \times \frac{11}{6} \div \frac{1}{4}$
3 Change division to multiplication and tip the next fraction (times and tip). Cancel where appropriate.	$= \frac{\cancel{6}^{1}}{7} \times \frac{11}{\cancel{6}_{1}} \times \frac{4}{1}$
4 Multiply and simplify.	$= \frac{44}{7}$
	$= 6\frac{2}{7}$

Graphics Calculator tip! Mixed operations with fractions

The calculation in worked example 29 can be entered as

$\boxed{6}$ $\boxed{a\frac{b}{c}}$ $\boxed{7}$ $\boxed{\times}$ $\boxed{1}$ $\boxed{a\frac{b}{c}}$ $\boxed{5}$ $\boxed{a\frac{b}{c}}$ $\boxed{6}$ $\boxed{\div}$ $\boxed{1}$ $\boxed{a\frac{b}{c}}$ $\boxed{4}$ then press $\boxed{\text{EXE}}$.



remember

The order of operations is the same for fractions as it is for whole numbers:
1. grouping symbols
2. multiplication and division (left to right)
3. addition and subtraction (left to right).

EXERCISE 5G Mixed operations with fractions

1 Find the following.

a $\frac{5}{8} + \frac{5}{6}$ **b** $\frac{7}{8} \div \frac{21}{32}$ **c** $\frac{3}{4} \times \frac{9}{10}$

d $\frac{4}{5} - \frac{3}{8}$ **e** $1\frac{3}{5} + 9\frac{4}{7}$ **f** $6\frac{1}{3} \div 4\frac{1}{2}$

g $1\frac{4}{5} \times \frac{9}{7}$ **h** $\frac{5}{2} - 1\frac{7}{8}$ **i** $5\frac{1}{4} - 2\frac{1}{5}$

WORKED Example 29

2 Find the following.

a $\frac{3}{5} \times 2\frac{1}{3} \div \frac{1}{2}$ **b** $1\frac{3}{4} \times 5 \div 3\frac{1}{2}$ **c** $4\frac{9}{10} + 2\frac{1}{4} - \frac{3}{5}$

d $5\frac{1}{3} + 7\frac{3}{8} - 8\frac{1}{2}$ **e** $2\frac{5}{6} \times 2 - 3\frac{1}{2}$ **f** $\frac{1}{2} + \frac{2}{3} \times \frac{1}{5}$

g $\frac{4}{5} \times 10 + 2\frac{3}{4}$ **h** $\frac{1}{3} + \frac{2}{9} \times \frac{3}{4}$ **i** $\frac{11}{12} - \frac{1}{2} \div \frac{3}{4}$

3 Calculate these, remembering to remove the grouping symbols first.

a $(\frac{5}{4} + \frac{2}{3}) \times \frac{1}{2}$ **b** $(5 - 3\frac{6}{7}) \times 4\frac{1}{3}$ **c** $(1\frac{1}{2} + \frac{3}{4}) \times (5 - 3\frac{3}{4})$

d $6 - (7\frac{5}{6} \div 2)$ **e** $\frac{3}{2} \div (4\frac{1}{2} + \frac{5}{6})$ **f** $\frac{1}{11} \times (8\frac{2}{9} - 5\frac{1}{6})$

g $(\frac{2}{3} \times 4\frac{1}{12}) - (7\frac{1}{2} \times \frac{1}{10})$ **h** $4 \times (2 - 1\frac{3}{7})$ **i** $6 \div (3\frac{2}{3} + 2\frac{7}{12})$

j $(\frac{2}{3} - \frac{1}{6}) \div (3\frac{1}{8} - 2\frac{5}{6})$

1 Without using a calculator find three-quarters of four-ninths of 78.

2 Without using a calculator find the answer to $\dfrac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{4} + \frac{1}{9}}$.

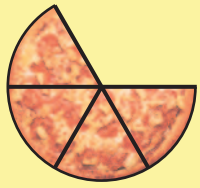
3 The fraction $\frac{1}{4}$ is a third of what number?

4 According to a will, a $120 000 estate is to be left to three children. Sam is to receive $\frac{2}{5}$ of the money, Georgia is to receive $\frac{1}{3}$ of the money and Olivia is to receive the remainder of the money. How much money is Olivia to receive?

5 When any positive number is multiplied by a number between zero and 1, what is the result?

You want more pizza?

Do you remember the question posed at the start of this chapter? Martin eats 2 slices of a pepperoni pizza which has been cut into 6 equal pieces while Olivia eats 3 pieces of the sausage pizza which has been cut into 8 equal pieces. Who has eaten more?



1. Try this question again, showing your calculations.
2. How many pieces of each pizza is left?
3. What fraction of each pizza is left?
4. What is the total amount of pizza left?

Daniella takes a slice of pepperoni pizza and Christopher takes a slice of sausage pizza. However, Christopher complains that his piece is smaller than Daniella's.

5. What is the difference in size between the larger and the smaller slice? (Write your answer as a fraction of a whole pizza.)

Daniella continues to eat the pepperoni pizza and Christopher continues to eat the sausage pizza.

6. How many slices of pizza should Daniella and Christopher each have so as to eat the same amount?
7. What fraction of each pizza is now left over?

Extension: Powers and square roots of fractions

Fractions and mixed numerals can be squared (index or power of 2) or cubed (index or power of 3) by using index notation just as with whole numbers.

For example, $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2^2}{3^2} = \frac{4}{9}$.

WORKED Example 30

Find: **a** $\left(\frac{1}{4}\right)^2$ **b** $\left(\frac{2}{5}\right)^3$ **c** $(1\frac{1}{3})^2$.

THINK

a 1 Write the question.

2 Square the numerator and the denominator.

3 Simplify.

WRITE

a $\left(\frac{1}{4}\right)^2$

$= \frac{1^2}{4^2}$

$= \frac{1}{16}$

THINK	WRITE
b 1 Write the question.	**b** $\left(\frac{2}{5}\right)^3$
2 Cube the numerator and the denominator.	$= \frac{2^3}{5^3}$
	$= \frac{2 \times 2 \times 2}{5 \times 5 \times 5}$
3 Simplify.	$= \frac{8}{125}$
c 1 Write the question.	**c** $(1\frac{1}{3})^2$
2 Change the mixed numeral to an improper fraction.	$= \left(\frac{4}{3}\right)^2$
3 Square both the numerator and the denominator.	$= \frac{4^2}{3^2}$
	$= \frac{16}{9}$
4 Change back to a mixed numeral.	$= 1\frac{7}{9}$

It is also possible to find the square roots of fractions and mixed numerals.

WORKED Example 31

Find: **a** $\sqrt{\frac{9}{16}}$ **b** $\sqrt{6\frac{1}{4}}$.

THINK	WRITE
a 1 Write the question.	**a** $\sqrt{\frac{9}{16}}$
2 Write the numerator and denominator as separate square roots.	$= \frac{\sqrt{9}}{\sqrt{16}}$
3 Calculate the square root of both the numerator and the denominator.	$= \frac{3}{4}$
b 1 Write the question.	**b** $\sqrt{6\frac{1}{4}}$
2 Change the mixed numeral to an improper fraction.	$= \sqrt{\frac{25}{4}}$
3 Write the numerator and the denominator as separate square roots.	$= \frac{\sqrt{25}}{\sqrt{4}}$
4 Calculate the square root of both the numerator and the denominator.	$= \frac{5}{2}$
5 Change to a mixed numeral.	$= 2\frac{1}{2}$

Graphics Calculator tip!

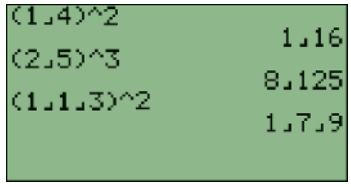
Powers and square roots

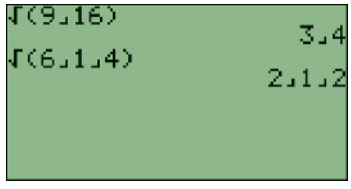
Powers and square roots

The calculator can be used to find different powers and square roots of fractions. To enter the power we use the (^) key or if we are squaring we can also use the (x^2) key. Remember to use grouping symbols around the fraction that is being used and (F ↔ D) to convert a decimal back to a fraction.

The screen at top right shows the calculations for worked example 30.

To obtain the square root of a fraction, first enter the square root sign by pressing (SHIFT) [$\sqrt{}$], then follow with grouping symbols and the fraction. The screen obtained for worked example 31 is shown at right.





remember

1. To square a fraction, square both numerator and denominator (base is raised to an index of 2).
2. To cube a fraction, cube both numerator and denominator (base is raised to an index of 3).
3. To find the square root of a fraction, take the square root of both numerator and denominator.
4. To find the square root of a mixed numeral, change the mixed numeral to an improper fraction, then find the square root of both the numerator and the denominator.

EXERCISE 5H

Extension: Powers and square roots of fractions

WORKED Example 30a

1 Find:

a $\left(\frac{1}{2}\right)^2$ b $\left(\frac{1}{3}\right)^2$ c $\left(\frac{2}{5}\right)^2$

d $\left(\frac{5}{6}\right)^2$ e $\left(\frac{3}{4}\right)^2$ f $\left(\frac{7}{10}\right)^2$

g $\left(\frac{5}{12}\right)^2$ h $\left(\frac{3}{11}\right)^2$ i $\left(\frac{3}{8}\right)^2$

WORKED Example 30b

2 Find:

a $\left(\frac{1}{4}\right)^3$ b $\left(\frac{1}{2}\right)^3$ c $\left(\frac{2}{5}\right)^3$

d $\left(\frac{3}{10}\right)^3$ e $\left(\frac{2}{3}\right)^3$ f $\left(\frac{1}{6}\right)^3$

Squaring and cubing fractions

WORKED Example 30c

3 Find:

a $(1\frac{1}{3})^2$ **b** $(2\frac{1}{4})^2$ **c** $(1\frac{1}{2})^2$

d $(1\frac{1}{10})^2$ **e** $(2\frac{1}{5})^2$ **f** $(3\frac{2}{3})^2$

WORKED Example 31

4 Find:

a $\sqrt{\frac{16}{25}}$ **b** $\sqrt{\frac{49}{100}}$ **c** $\sqrt{\frac{9}{64}}$

d $\sqrt{\frac{25}{36}}$ **e** $\sqrt{2\frac{1}{4}}$ **f** $\sqrt{1\frac{7}{9}}$

g $\sqrt{1\frac{9}{16}}$ **h** $\sqrt{2\frac{14}{25}}$ **i** $\sqrt{1\frac{15}{49}}$

Mathcad: Square root of a fraction

5 Calculate:

a $\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2$ **b** $\left(\frac{2}{3}\right)^2 - \left(\frac{1}{2}\right)^2$ **c** $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{9}\right)^2$

d $\left(\frac{4}{7}\right)^2 \div \left(\frac{3}{7}\right)^2$ **e** $\sqrt{\frac{4}{9}} + \left(\frac{2}{3}\right)^2 - \frac{1}{2}$ **f** $\sqrt{\frac{16}{25}} \times \left(\frac{1}{2}\right)^2 - \frac{1}{10}$

WORKSHEET 5.3

MATHS QUEST CHALLENGE

1 Find the missing power for this calculation: $\left(\frac{3}{5}\right)^{-} = \frac{2187}{78\,125}$.

2 A length of rope is cut in half and one half is used. Then one third of the other half is cut off and used. If the remaining piece of rope is 10 m long, how long was the original rope?

3 Solve this expression.

$$\frac{\left(\frac{1}{2}\right)^3}{\left(1 + \frac{1}{1+3}\right)^2}$$

4 Look for a pattern in this list of fractions and write down the next two in the sequence.

$\frac{2}{3} \rightarrow \frac{9}{5} \rightarrow \frac{16}{14} \rightarrow \frac{23}{30} \rightarrow \frac{30}{53}$

summary

Copy the sentences below. Fill in the gaps by choosing the correct word or expression from the word list that follows.

1 The _____ is the top part of the fraction.

2 The _____ is the bottom part of the fraction.

3 Different fractions can be compared or ordered by writing each one with the _____ denominator.

4 _____ order is from smallest to largest.

5 _____ order is from largest to smallest.

6 _____ are found by multiplying or dividing both the numerator and the denominator by the same number as long as the number is not equal to zero.

7 To _____ fractions, find the largest number that is a factor of the numerator and the denominator and divide both by that number.

8 _____ fractions can be changed to mixed numerals by dividing the numerator by the denominator. The _____ becomes the numerator and the denominator remains the same.

9 Mixed _____ can be changed to improper fractions by multiplying the whole number and the denominator then adding the numerator. The result becomes the numerator and the denominator remains the same.

10 Fractions must have the same denominator to be added or subtracted. This is done by finding a _____ denominator. Once the denominators are the same, the numerators can be added or subtracted.

11 To multiply fractions, simply _____ the numerators and multiply the denominators. It is often easier to _____ or simplify numerators and denominators before multiplying. Remember to change mixed numerals to improper fractions first.

12 To find the _____ of a number, swap the numerator and denominator.

13 To divide fractions, change the ÷ to × and _____ the second fraction. Remember to change _____ numerals to improper fractions first.

WORD LIST

denominator	common	descending
ascending	same	remainder
mixed	equivalent fractions	numerals
tip	cancel	reciprocal
multiply	numerator	
improper	simplify	

CHAPTER review

1 What fraction of the eggs in this carton are brown?

5A

2 Write three equivalent fractions for the following.

a $\frac{3}{8}$ b $\frac{4}{9}$ c $\frac{5}{11}$ d $\frac{6}{7}$

5A

3 Fill the gaps to make equivalent fractions.

a $\frac{1}{2} = \frac{\square}{4} = \frac{3}{\square}$ b $\frac{1}{3} = \frac{3}{\square} = \frac{\square}{15}$ c $\frac{3}{4} = \frac{9}{\square} = \frac{\square}{28}$

d $\frac{2}{5} = \frac{\square}{10} = \frac{16}{\square}$ e $\frac{1}{4} = \frac{5}{\square} = \frac{\square}{100}$ f $\frac{3}{10} = \frac{\square}{100} = \frac{300}{\square}$

5A

4 Simplify these fractions.

a $\frac{42}{49}$ b $\frac{81}{108}$ c $\frac{18}{20}$ d $\frac{21}{28}$

e $\frac{36}{72}$ f $\frac{50}{150}$ g $\frac{75}{125}$ h $\frac{2}{80}$

5B

5 Simplify the following mixed numerals.

a $6\frac{8}{10}$ b $3\frac{6}{9}$ c $4\frac{15}{20}$ d $1\frac{20}{100}$

5B

6 What fraction of the months of the year begin with the letter J? Express your answer in simplest form.

5B

7 Convert these improper fractions to mixed numerals.

a $\frac{16}{3}$ b $\frac{21}{5}$ c $\frac{12}{7}$ d $\frac{11}{2}$

e $\frac{5}{4}$ f $\frac{80}{7}$ g $\frac{55}{9}$ h $\frac{72}{10}$

5C

8 Express these mixed numerals as improper fractions.

a $2\frac{3}{4}$ b $9\frac{7}{8}$ c $3\frac{5}{7}$ d $5\frac{5}{6}$

e $3\frac{2}{3}$ f $9\frac{9}{10}$ g $3\frac{11}{12}$ h $8\frac{1}{2}$

5C

9 Find:

a $\frac{5}{6} + \frac{1}{8}$ b $\frac{7}{9} - \frac{2}{5}$ c $\frac{12}{5} - \frac{4}{3}$

d $\frac{5}{4} + \frac{11}{2}$ e $1\frac{7}{8} + 2\frac{5}{6}$ f $10\frac{3}{7} - 8\frac{1}{2}$

g $6\frac{1}{3} - 2\frac{5}{9}$ h $5\frac{1}{2} - 2\frac{5}{6}$

5D

10 Find:

a $1\frac{1}{5} + 3\frac{1}{3} - \frac{13}{15}$ b $4\frac{7}{9} - 1\frac{5}{6} + 3\frac{1}{3}$

5D

5D

11 Tony caught 3 fish with masses:
$2\frac{7}{10}$ kg, $2\frac{1}{2}$ kg and $2\frac{4}{5}$ kg.

a Which fish is the heaviest?
b What is the combined mass of the fish?
c What is the difference in mass between the heaviest and the lightest fish?

5E

12 Find:

a $\frac{4}{7} \times \frac{5}{9}$
b $\frac{2}{5} \times \frac{3}{5}$
c $\frac{25}{36} \times \frac{18}{20}$
d $\frac{56}{81} \times \frac{18}{64}$
e $4\frac{2}{5} \times 3\frac{2}{11}$
f $10\frac{2}{7} \times 6\frac{2}{9}$
g $\frac{4}{5}$ of 55
h $\frac{2}{11}$ of 121
i $\frac{7}{9}$ of $\frac{18}{84}$
j $\frac{1}{12}$ of 200

5E

13 Find:

a $\frac{3}{5} \times \frac{5}{6} \times \frac{2}{7}$
b $\frac{3}{4} \times \frac{16}{21} \times \frac{7}{20}$
c $\frac{5}{11} \times \frac{3}{10} \times 1\frac{1}{10}$
d $\frac{4}{9} \times 2\frac{1}{4} \times \frac{3}{5}$
e $3\frac{5}{12} \times 1\frac{3}{5} \times 2\frac{1}{2}$
f $\frac{6}{17} \times 1\frac{1}{4} \times 3\frac{2}{5}$

5F

14 Write the reciprocals of:

a $\frac{5}{2}$
b $\frac{2}{7}$
c $\frac{1}{9}$
d 12
e 124
f $\frac{1}{4}$
g $5\frac{3}{4}$
h $8\frac{1}{2}$
i $7\frac{2}{5}$
j $2\frac{4}{9}$
k $3\frac{5}{8}$
l $4\frac{1}{5}$

5F

15 Find:

a $\frac{4}{5} \div \frac{8}{15}$
b $\frac{9}{10} \div \frac{27}{40}$
c $\frac{7}{9} \div 4$
d $\frac{3}{7} \div 10$
e $1\frac{1}{2} \div 4\frac{4}{5}$
f $6\frac{2}{3} \div 4\frac{1}{6}$

5G

16 Calculate:

a $(\frac{4}{9} + \frac{5}{6}) \div 3$
b $\frac{5}{14} \times (\frac{3}{10} + \frac{2}{5})$
c $(\frac{2}{3} - \frac{1}{6}) \times (\frac{3}{4} + \frac{5}{8})$
d $\frac{3}{4} \div \frac{9}{10} + \frac{5}{7} \times \frac{14}{15}$
e $2 + 4\frac{1}{2} \div 2\frac{1}{4}$
f $7 - 2\frac{1}{4} \div 1\frac{1}{2}$

5H

17 Calculate:

a $\left(\frac{4}{5}\right)^2$
b $\left(\frac{9}{10}\right)^2$
c $\left(\frac{3}{4}\right)^2$
d $\left(\frac{7}{8}\right)^2$
e $\left(\frac{1}{3}\right)^3$
f $\left(\frac{5}{6}\right)^3$
g $(1\frac{1}{2})^2$
h $(2\frac{1}{4})^2$
i $(1\frac{3}{5})^2$

5H

18 Calculate:

a $\sqrt{\frac{81}{100}}$
b $\sqrt{\frac{25}{49}}$
c $\sqrt{2\frac{7}{9}}$
d $\sqrt{2\frac{9}{36}}$

test yourself CHAPTER 5