10

Directed numbers and the number plane

A group of mountain climbers were climbing one of the highest mountains in the world. From their base camp, the group travelled 506.3 m upwards. One climber started to suffer altitude sickness and was escorted down by another climber to a point 273.1 m below the base camp. How far apart were the two groups?

This chapter deals with directed numbers; that is, numbers with a direction as well as size.

Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest 7* CD-ROM or ask your teacher for a copy.

SkillSHEET 10.1

1 Complete each statement by inserting the correct symbol, < or >.

a 5 1 **b** 17 71 **c** 0 10

SkillSHEET 10.2

2 **a** Arrange the following in ascending order: 5, 120, 0, 3, 15.

b Arrange the following in descending order: 25, 19, 42, 4, 24.

SkillSHEET 10.3

3 Draw a number line with 10 equal intervals marked from 0 to 10. Mark the following numbers on the number line with a dot.

a 3 **b** 0 **c** 8

SkillSHEET 10.4

4 Refer to the number line drawn for question **3**. Which two numbers are:

a 1 unit away from 3? **b** 2 units away from 8?

SkillSHEET 10.5

5 Evaluate the following.

a 3^2 **b** 2^3 **c** $\sqrt[3]{27}$

SkillSHEET 10.6

6 Calculate the following using the correct order of operations.

a $5 + 2 \times 7$ **b** $18 \div 3 \times 2$ **c** $3^2 - 4 \times 2$

SkillSHEET 10.7

7 Calculate each of the following.

a $\frac{1}{3} + \frac{1}{2}$ **b** $\frac{3}{5} - \frac{2}{7}$ **c** $2\frac{3}{4} + 1\frac{2}{3}$

d $\frac{5}{6} \times \frac{3}{7}$ **e** $\frac{3}{8} \div \frac{1}{4}$ **f** $3\frac{2}{5} \div 2\frac{1}{2}$

SkillSHEET 10.8

8 Calculate each of the following.

a $0.5 + 0.26$ **b** $2.73 - 1.49$ **c** 0.3×0.2

d $(0.4)^2$ **e** $4.8 \div 0.5$ **f** $0.032 \div 0.04$

Integers on the number line

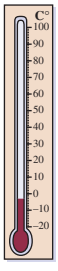
Using positive and negative numbers in our daily lives

Most people understand the concept of negative numbers as soon as they are able to use money. Does the following example sound familiar?

The phone rings and Deb is arranging to go to the city with her friends. Having spent all her money on Christmas presents, she borrows $15 from her mother: $10 for the movies and $5 for snacks. Once she has spent this, she has *less* than $0. She owes her mother $15, which she must pay back when she has earned some more money.

The concept of owing money can be expressed as a number less than zero or a negative number. The number of dollars that Deb has after her trip to the city is –15. If she had spent nothing on snacks and returned the $5 to her mother, she would have had –$10.

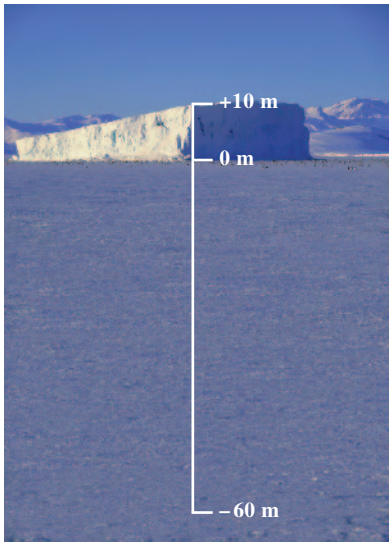
Other ways that negative numbers can be used are:



The temperature was –3°C last night or 3 degrees below zero.

My bank statement showed that I had withdrawn more than I had deposited so I was $5 in the red. I had –$5 or I owed the bank $5.

Amount $		
Deposits	**Withdrawals**	**Balance**
250		250
180		430
	35	395
	400	5–



An iceberg which extends further below sea level than it does above sea level may extend from 10 m to –60 m.

Positive and negative numbers have both *size* and *direction*, so they are often called *directed numbers.*

Positive whole numbers, negative whole numbers and zero are all integers.

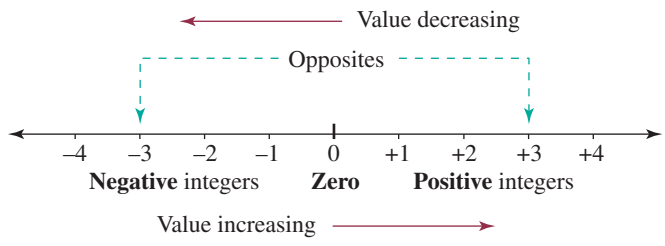
Integers can be represented as –3, –2, –1, 0, +1, +2, +3.
When we write –3, we read it as negative three. When we write +2, we read it as positive two.

WORKED Example 1

Write an integer suggested by each of the following examples.
a The temperature was 1 degree below zero.
b Kate has $500 in her bank account.

THINK	WRITE
a Below zero refers to negative.	**a** –1
b She actually has money so the integer is positive.	**b** +500

Integers can be represented on a number line.



With the exception of zero, each integer has an opposite. For example, in the number line, the integer –3 may be read as the *opposite of* +3. Therefore, –(–2) becomes the opposite of negative 2; that is, positive 2.

Positive or negative *direction* symbols are placed to the left of the first digit of the number and are *not spaced*, whereas *operation* symbols for addition and subtraction are *spaced* between numbers. For example, 3 + 2 = +5.

The number line

The number line can be used when comparing integers and when adding and subtracting integers.

Zero is neither positive nor negative. It is the basic reference value on the number line. So +3 is three units to the right of zero and –3 is three units to the left. The further a number is to the right on a number line, the greater the value; the further a number is to the left on a number line, the smaller the value.

Symbols and their meanings

> means 'is greater than' **≥ means 'is greater than or equal to'**
< means 'is less than' **≤ means 'is less than or equal to'**

WORKED Example 2

Complete each statement by inserting the correct symbol: >, < or =.

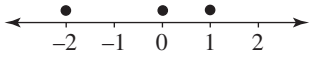
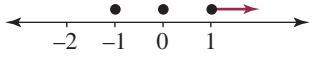
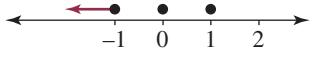
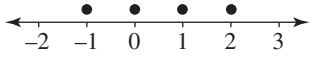
a 2 5 **b** –4 –1 **c** 0 –3 **d** 6 –2

THINK	WRITE
a 2 is to the left of 5 on the number line, so 2 is smaller.	**a** $2 < 5$
b –4 is to the left of –1 on the number line, so –4 is smaller.	**b** $-4 < -1$
c 0 is to the right of –3 on the number line, so 0 is larger.	**c** $0 > -3$
d 6 is to the right of –2 on the number line, so 6 is larger.	**d** $6 > -2$

WORKED Example 3

Graph the following sets of integers on the number line.

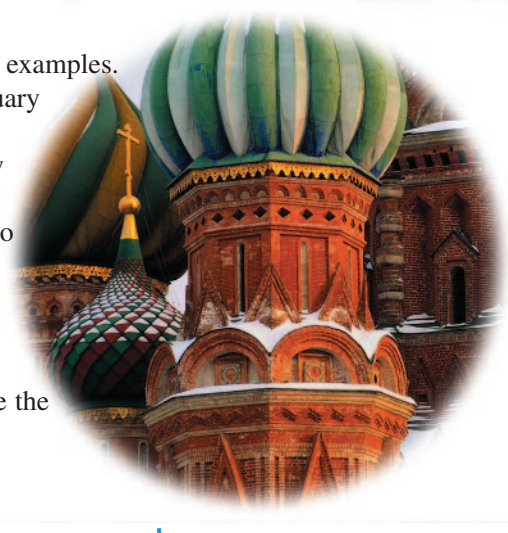
a –2, 0, 1 **b** integers > –2 **c** integers ≤ 1 **d** integers between –2 and 3

THINK	WRITE
a These are individual integers, so mark each one with a solid dot.	**a** 
b The symbol > means 'is greater than or to the right of –2', so this is an infinite list not including –2.	**b** 
c The symbol ≤ means 'is less than or equal to, or to the left of 1', so this is an infinite list including 1.	**c** 
d Between –2 and 3 means 'not including the boundary numbers –2 and 3'.	**d** 

remember

1. Integers are positive whole numbers, negative whole numbers and zero.
2. Integers, including zero, are *directed* numbers.
3. On a number line, *positive* integers are to the right of zero and *negative* integers are to the left of zero.
4. Symbols and their meanings:
   > means 'is greater than' ≥ means 'is greater than or equal to'
   < means 'is less than' ≤ means 'is less than or equal to'.

EXERCISE 10A Integers on the number line



WORKED Example 1

1 Write an integer suggested by each of the following examples.

a The minimum temperature in Moscow on 1 January 2000 was 5 degrees below zero.
b The maximum temperature in Jerusalem on New Year's Day 2000 was a mild 23°C.
c Naomi had no money left so she borrowed \$20 to spend at Chatswood.
d Sila's bank statement showed that he was in the red by \$150.
e Luis deposited \$20 into his account.
f Our unit at Surfer's Paradise was 11 floors above the ground floor.
g The lift went 2 floors below ground level.

2 Which of the following are integers?

a -6 **b** 32 **c** $\frac{2}{5}$ **d** 0.8
e $2\frac{1}{4}$ **f** 0 **g** -1000 **h** -3.4

Mathcad: Greater than or less than

3 Are the following true or false?

a The opposite of -4 is $+4$.
b Whole numbers are integers.
c Zero is not an integer.
d $-(-3) = +3$
e -5 is a negative integer.
f $2 > -2$
g $0 > -5$ **h** $-8 < -3$ **i** $-4 > -1$
j $0 < 3$ **k** $-5 = 5$ **l** $-3 < 0$

SkillSHEET 10.1

WORKED Example 2

4 Complete each statement by inserting the correct symbol >, < or = :

a 3 8 **b** 61 16 **c** 90 500
d 110 700 **e** 24 26 **f** 30 3
g 4 20 **h** -9 9 **i** 0 -2
j -5 -2 **k** 6 0 **l** -100 1

SkillSHEET 10.2

5 Arrange each of the following in ascending order from smallest to largest.

a $-3, 2, 0, 1, -8$ **b** $-38, 3, -6, 0, 5$ **c** $-4, -8, -21, -1, 4$

6 Arrange each of the following in descending order from largest to smallest.

a $0, -5, 5, 3, -2$ **b** $-18, -20, -9, -1, 2$ **c** $33, -22, 10, -9, 20$

7 List the next three integers in each sequence.

a 4, 2, 0, _____, _____, _____
b $-3, -6, -9$, _____, _____, _____
c $-64, -32, -16$, _____, _____, _____

SkillSHEET 10.3

EXCEL Spreadsheet: The number line

8 Draw a number line with 10 equal intervals, marked from -5 to $+5$. Mark the following integers with a coloured dot.

a -3 **b** 0 **c** 2
d the integer four units to the right of -5

SkillSHEET 10.4

9 Draw a number line with 10 equal intervals, marked from -5 to $+5$. Which two integers are:

a 1 unit away from 3?
b 1 unit away from -3?
c 4 units away from -1?
d 2 units away from 2?

10 **multiple choice**

a The whole numbers between 18 and 23 are:
- **A** 19, 20, 21, 22, 23
- **B** 18, 19, 20, 21, 22, 23
- **C** 18, 19, 20, 21, 22
- **D** 19, 20, 21, 22

b The whole numbers greater than 5 (>5) are:
- **A** 5, 6, 7, 8, 9, . . .
- **B** 0, 1, 2, 3, 4
- **C** 6, 7, 8, 9
- **D** 6, 7, 8, 9, . . .

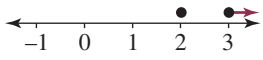
c The whole numbers less than or equal to 4 (≤ 4) are:
- **A** 0, 1, 2, 3, 4
- **B** 4, 5, 6, 7, 8, 9, . . .
- **C** 0, 1, 2, 3
- **D** 5, 6, 7, 8, 9, . . .

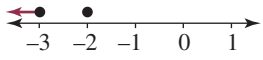
WORKED Example 3

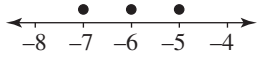
11 Graph the following sets of integers on different number lines:

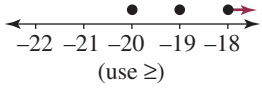
a 0, 3, 6
b integers > 5
c integers < 3
d integers ≥ 9
e integers between 8 and 12
f integers < –2
g integers ≥ –6
h integers between –8 and –2
i integers between –2 and 2
j integers > –3
k the negative integers
l the non-negative integers.

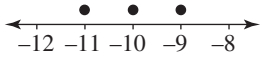
12 List the integers graphed on each number line.

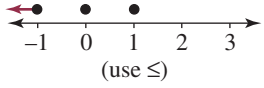
a


b


c


d


e


f


Comparing temperatures

Consider the following temperatures:

a Ethylene glycol is used as antifreeze in car radiators. This chemical lowers the freezing point of the coolant to –15°C.

b Azizia in Libya recorded a temperature of 58°C.

c Pure water freezes at 0°C.

d Pure water boils at 100°C.

e The average annual temperature in Antarctica is –49°C.

f The world's lowest temperature of –86.6°C was recorded at Vostok in Antarctica.

1. List the temperatures that are negative.
2. List the temperatures that are:
   - **i** greater than 20°C
   - **ii** less than –20°C
   - **iii** greater than –30°C
   - **iv** less than –40°C
   - **v** between –10°C and –60°C.
3. True or false? The average temperature in Antarctica is greater than the freezing point of coolant in a car radiator with antifreeze added.

Locate the letter at each position on the number lines below to find the code to answer the questions given.

What is shish-kabob?

A	C	D	E	F	I	K	N	O	R	S	T	W
-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6

-2	+2	+2	-4	-5	+2	+2	0	-3	-4	+2	+1	-6
+4	0	-3	+6	-3	+3	+2	+3	+4	+5	-1	-5	0

Where is the Yukon?

E	N	C	Y	I	T	A	R	H	D	O
-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2

-6	-7	-12	-5	-5	-8	-7	-2	-5	-9	-8	-11
-11	-2	-5	-7	-4	-12	-5	-11	-10	-6	-11	-6	-3	-6

Are whales fish?

+1	H	-7	
0	O	0	-2
-1	T	-1	-3
-2	M	+1	-2
-3	A	-9	-2
-4	Y	-4	-3
-5	L	-3	-5
-6	S	-8	-6
-7	N	-9	
-8	R		
-9	E		

How does a cricket chirp?

O	N	R	I	H	S	G	T	E	U	B	W	Y
-1	0	-2	+1	+2	-3	+3	-4	+4	-5	+5	-6	-7

+5	-7	-2	-5	+5	+5	+1	0	+3
+1	-4	-3	-6	+1	0	+3	-3	
-4	-1	+3	+4	-4	+2	+4	-2	

What were the first wheeled vehicles?

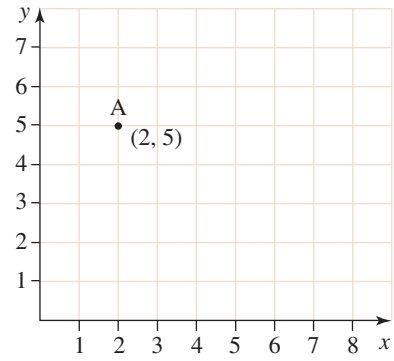
T	S	R	O	I	H	C	A
-9	-8	-7	-6	-5	-4	-3	-2

-3	-4	-2	-7	-5	-6	-9	-8

Positive integers and zero on the number plane

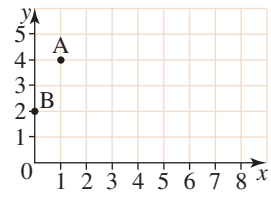
The number plane (or Cartesian plane) has 2 axes: the *horizontal* or *x-axis* and the *vertical* or *y-axis*.

Each point on the number plane is described by its position relative to the *x*- and *y*-axes. A pair of *coordinates* or an ordered pair (*x*, *y*) fix the position of the point, where *x* units is the distance along the *x*-axis and *y* units is the distance along the *y*-axis. So the point marked A has coordinates (2, 5) because the point is 2 units to the right of zero along the *x*-axis and 5 units up the *y*-axis from zero.



WORKED Example 4

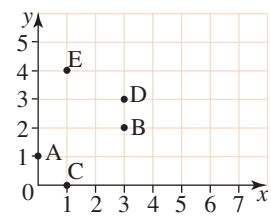
In the number plane at right, find the coordinates of: a A b B.



THINK	WRITE
a Point A is 1 unit to the right of zero along the *x*-axis and 4 units up the *y*-axis from zero.	**a** A (1, 4)
b Point B is on the *y*-axis, so is 0 units to the right of zero along the *x*-axis and is 2 units up the *y*-axis from zero.	**b** B (0, 2)

WORKED Example 5

Use a letter to name the points on this number plane which have the following coordinates.
a (3, 3) b (1, 0)



THINK	WRITE
a The point with coordinate (3, 3) is 3 units to the right of zero along the *x*-axis and 3 units up the *y*-axis.	**a** (3, 3) is D
b The coordinate (1, 0) is 1 unit to the right of zero along the *x*-axis and 0 units up the *y*-axis.	**b** (1, 0) is C

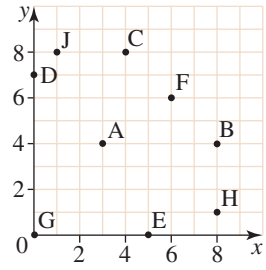
remember

1. The x-axis is the horizontal axis and the y-axis is the vertical axis.
2. An ordered pair (x, y) fixes the position of a point on the number plane, where x units is the distance along the x-axis and y units is the distance along the y-axis.
3. The x-coordinate indicates the number of units from zero that the point is along the x-axis and the y-coordinate indicates the number of units up the y-axis from zero.

EXERCISE 10B Positive integers and zero on the number plane

Mathcad Number plane (positive axis)

Questions **1** to **3** refer to the diagram shown at right.



WORKED Example 4

1 Find the coordinates of the following.

a A **b** B **c** C
d D **e** E

WORKED Example 5

2 Write a letter to name each of the points with the following coordinates.

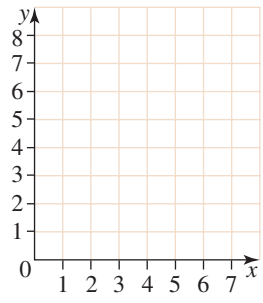
a (0, 0) **b** (6, 6) **c** (8, 1)
d (1, 8)

3 What do points A and B have in common?

4 Draw axes as shown and plot the following points in the order given, joining each point to the next one.

(0, 0) (2, 8) (2, 0) (0, 3) (4, 6) (0, 0)

What shape do they form?



MATHS QUEST CHALLENGE

Here is a game to try with a partner! The first person chooses a number from the list 1, 2, 3, 4, 5 and 6. The second player also chooses a number from this list and adds it to the first number chosen. The first player chooses again (any number from the list can be used even if it has been used before) and adds this to the total.

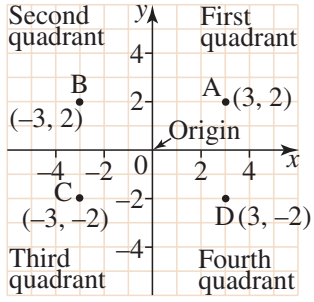
Players take turns selecting a number and adding it to the total until a total of 51 is reached. The player who chose the last number to make it 51 is the winner.

Play this game a number of times. Are there any strategies you can use to help you win the game?

Integers on the number plane

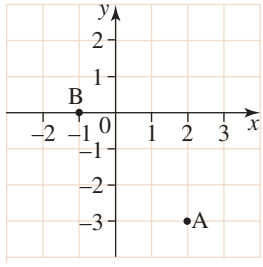
The axes on the number plane can be extended to include negative integers as shown.

The *origin* (0, 0) is the point where the two axes intersect, dividing the plane into four *quadrants*. The quadrants are numbered in an anticlockwise direction, beginning in the top right quadrant.



WORKED Example 6

Write the coordinates and state the quadrant or axis of each point on this number plane.



THINK	WRITE
a 1 A is 2 units to the right of the origin along the x-axis and is 3 units down from the origin along the y-axis.	**a** A (2, –3)
2 It is in the lower right-hand corner.	A is in the fourth quadrant.
b 1 B is 1 unit to the left of the origin along the x-axis and is 0 units up or down from the origin along the y-axis.	**b** B (–1, 0)
2 It is on the x-axis.	B is on the x-axis.

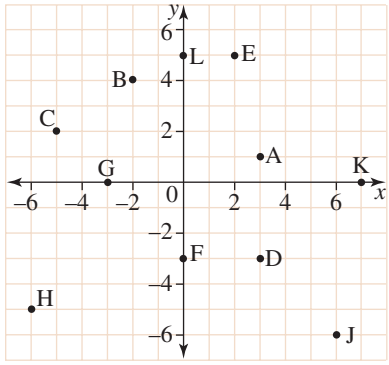
remember

1. The horizontal number line is the x-axis, and the vertical number line is the y-axis.
2. Points are represented by ordered pairs of integers (x, y).
3. The origin is the point (0, 0), where the axes intersect.
4. Each quarter is called a *quadrant* and is numbered from one to four in an anti-clockwise direction beginning with the top right-hand quadrant.

EXERCISE 10C Integers on the number plane

Mathcad Number plane

Use the diagram shown here to find the answers to questions **1** to **9**.



WORKED Example 6

1 Write the coordinates and state the quadrant or axis of each point.
a A **b** B **c** H **d** F **e** J

2 Name the point and give its quadrant or axis.
a (–5, 2) **b** (0, 5) **c** (3, –3) **d** (2, 5) **e** (–3, 0)

3 Give the *x*-coordinate of the following.
a A **b** D **c** K **d** L

4 Give the *y*-coordinate of the following.
a C **b** J **c** G **d** F

5 What is the *x*-value of all points on the *y*-axis?

6 What is the *y*-value of all points on the *x*-axis?

7 List all points lying in the third quadrant in the diagram.

8 In which quadrant do all points show the sign pattern (+ , –)?

9 True or false?
a F and D have the same *y*-coordinate.
b A and D have the same *x*-coordinate.
c The origin has coordinates (0, 0).
d The point at (3, 5) is the same point as (5, 3).
e The point at (–5, 4) is in the third quadrant.
f The point at (0, 2) must lie on the *y*-axis.

10 **multiple choice**

a The point (5, –2) lies:
A in the first quadrant **B** in the second quadrant
C in the third quadrant **D** in the fourth quadrant

b The point at (0, 4) lies:
A in the first quadrant **B** on the *y*-axis
C in the second quadrant **D** on the *x*-axis

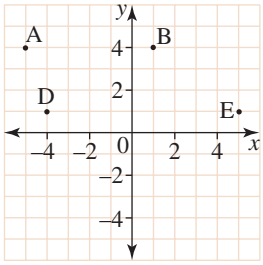
c The point (–4, 5) lies:

A in the first quadrant
B in the second quadrant
C in the third quadrant
D in the fourth quadrant

11 Draw up a number plane with both axes scaled from –6 to 6. Plot the points listed and join them with straight lines in the order given. Name the completed shape.

a (5, 5), (3, 2), (–2, 2), (0, 5), (5, 5)
b (4, –1), (4, –5), (–1, –3), (4, –1)
c (–4, 4), (–2, 1), (–4, –5), (–6, 1), (–4, 4)
d (–2, 1), (1, 1), (1, –2), (–2, –2), (–2, 1)

12 **a** Find the coordinates of a point, C, so that ABCD is a parallelogram.
b Find the coordinates of a point, F, so that DBEF is a kite shape.
c List two points on the line joining D to E.
d Give the coordinates of a point, T, in the third quadrant, which would complete the isosceles triangle ADT.
e Give an example of a point that is two units from D.



WORKSHEET 10.1

10 QUICK QUESTIONS 1

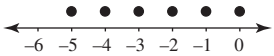
1 True or false? The number –2.5 is called an integer.

2 True or false? –6 < –2

3 List the integers between –11 and –7.

4 Arrange these numbers in ascending order: 7, 0, –3, 10, –15

5 Describe the integers graphed on the number line.



6 On a drawn number line with 8 equal intervals marked from –4 to +4, graph integers ≥ –3.

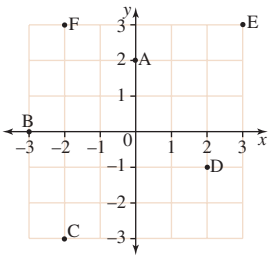
Use the following number plane diagram for questions **7**, **8** and **9**.

7 Give the coordinates and quadrant for the point C.

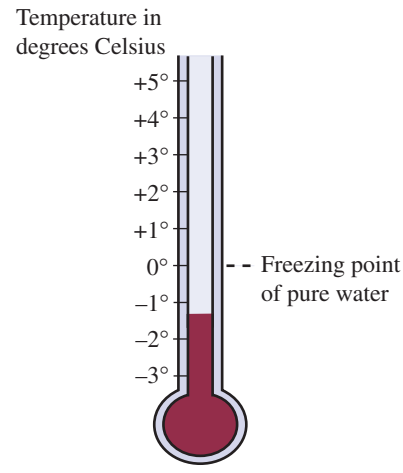
8 Name the point fitting the description (–2, 3).

9 True or false? Point D is in the second quadrant.

10 On a drawn number plane with both axes scaled from –5 to +5, plot the points listed and join them in the order given. Name the shape you have constructed.
(–5, 4), (2, 3), (4, –2), (–2, –3), (–5, 4)



Addition of integers



One cold and frosty morning the temperature is –3°C, but by 10 am it has risen to +4°C. Addition of integers can be used to find the temperature at 10 o'clock.

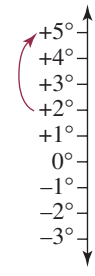
In the Celsius temperature scale, pure water freezes at 0°C and boils at 100°C. Human body temperature is around 37°C. Air temperatures are between 25°C and 35°C on hot days and between 18°C and 21°C on cool days. On frosty mornings and in snowy regions temperatures are *below zero*. Let us now experiment with adding integers that we find in temperature measurements.

Case 1: Adding positives

A rise in temperature of 3°C from 2°C

Starting at 2°C, an increase of 3°C results in a temperature of 5°C.

$$+2 + +3 = +5$$

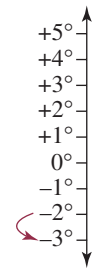


Case 2: Adding negatives

A fall in temperature of 1°C from –2°C

Starting at –2°C, a fall of 1°C results in a temperature of –3°C.

$$-2 + -1 = -3$$

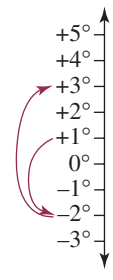


Case 3: Mixed signs

A rise in temperature of 5°C from –2°C

Starting at –2°C, a rise of 5°C results in a temperature of +3°C.

$$-2 + +5 = +3$$



A fall in temperature of 3°C from +1°C

Starting at +1°C, a fall of 3°C results in a temperature of –2°C.

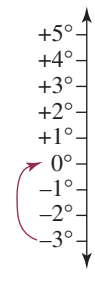
$$+1 + -3 = -2$$

Case 4: Adding opposites

A rise in temperature of 3°C from –3°C

Starting at –3°C, a rise of 3°C results in a temperature of 0°C.

$$-3 + +3 = 0$$



Note: Adding opposites always results in zero.

Assume that the number is positive if there is no sign.

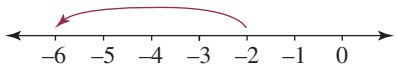
For example, –3 + +3 = 0 could be written as –3 + 3 = 0.

These examples of adding temperatures show us the general rules for adding integers: just use a number line or number sentences and add integers in the same way.

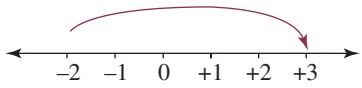
WORKED Example 7

Write number sentences to show the addition problems suggested by the following diagrams.

a



b



THINK

a Write the addition statement suggested by the diagram. (The first integer is 2 units to the *left* of zero, so start at –2. The second integer is 4 units to the *left* of –2, finishing at –6.) Find the answer.

b Write the addition statement suggested by the diagram. (The first integer is 2 units to the *left* of zero so start at –2. The second integer is 5 units to the *right* of –2, finishing at +3.) Find the answer.

WRITE

a $-2 + -4 = -6$

b $-2 + +5 = +3$

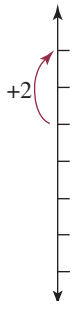
remember

1. To add integers, use a number line or number sentence.
2. Adding opposites always results in zero.
3. Assume that the number is positive if there is no sign.

EXERCISE 10D Addition of integers

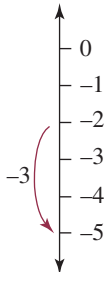
1 For each of the following temperature changes, write the sum suggested by the diagram and the resulting temperature. See cases 1 to 4 on page 364.

a



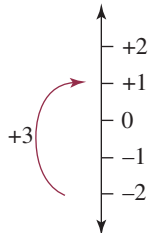
$+1 + +2 =$ __

b



$-2 + -3 =$ __

c



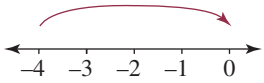
$-2 + +3 =$ __

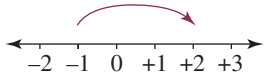
Mathcad Addition and subtraction of integers

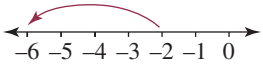
EXCEL Spreadsheet Addition of integers

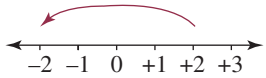
WORKED Example 7

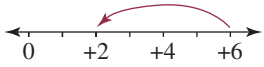
2 Write number sentences to show the addition problems suggested by the following diagrams.

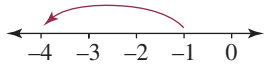
a


b


c


d


e


f


3 Copy and complete these number sentences. (Draw a number line if you wish.)

a 5 + –2 **b** –3 + –4 **c** –2 + 2
d 6 + –5 **e** –5 + 5 **f** 4 + –6
g –5 + 7 **h** 6 + –9 **i** –4 + 6
j 3 + –3 **k** –8 + –2 **l** 0 + –6

Remember: Rules for addition of integers are.

1. For *same signs*: *add* and keep the sign: –3 + –5 = –(3 + 5) = –8
2. For *different signs*: *subtract* the smaller number from the larger number and use the sign of the number further from zero –8 + 5 = –(8 – 5) = –3
3. For *opposites*: *opposite* integers add to zero –4 + 4 = 0

4 Write the answer for each of the following.

a –5 + –2 **b** –6 + 4 **c** –8 + 8
d 3 + –7 **e** –3 + 7 **f** –3 + –7
g –8 + 12 **h** 19 + –22 **i** –64 + –36
j –80 + 90 **k** –2 + 4 **l** –15 + –7

5 **a** Copy and complete the addition table shown below.
b What pattern is shown along the leading (dotted) diagonal?
c What pattern is shown along the other (unmarked) diagonal?
d Is the chart symmetrical about the leading diagonal?
e Comment on any other number patterns you can see.

+	–2	–1	0	1	2
–2		–3			
–1				0	
0	–2				
1					
2					

6 Copy and complete this addition table.

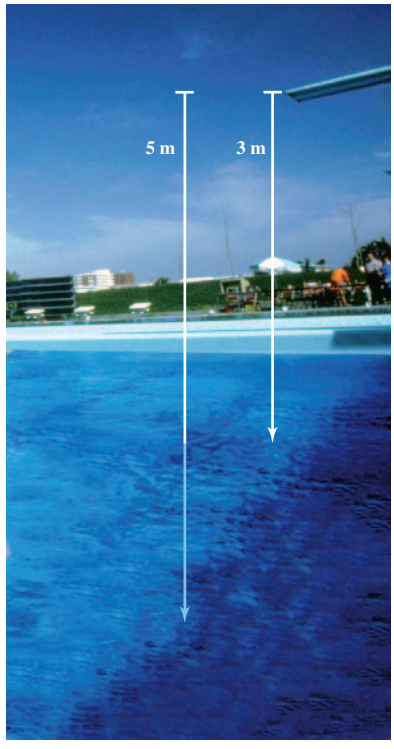
+	–13	5
21		
–18		

7 Write the number that is:

a 6 more than –2 **b** 5 more than –8 **c** 8° above –1°C
d 3° below 2°C **e** 3 to the right of –4 **f** 4 to the left of –3.

8 Model each situation with an integer number sentence that shows the result of the following.

a From ground level, a lift went down 2 floors, then down another 3 floors.
b From ground level, a lift went down 3 floors, then up 5 floors.
c From ground level, a lift went up 5 floors, then down 6 floors.
d Australia was 50 runs behind, then made another 63 runs.
e An Olympian dived down 5 metres from a board 3 metres above water level.
f At 5.00 pm in Thredbo the temperature was 1°C. It then fell 6 degrees by 11.00 pm.
g A submarine at sea level dived 50 metres, then rose 26 m.
h An account with a balance of $200 had $350 withdrawn from it.



9 **multiple choice**

From ground level, a lift went down 2 floors, then down another 3 floors to a level 5 floors below the ground. The number sentence that describes this situation is:

A $2 + 3 = 5$ **B** $-2 + -3 = -5$
C $-2 + 3 = 1$ **D** $2 + -3 = -1$

10 Describe a situation to fit each of the number sentences below.

a $-3 + -2 = -5$ **b** $-10 + -40 = -50$ **c** $2 + -6 = -4$
d $-20 + 20 = 0$ **e** $-8 + 10 = 2$

11 When Merv first notices the spider, it is sitting about 3 cm below a light switch on the wall. He watches the spider move about 12 cm up the wall then 5 cm down where it stops for a few seconds. The spider then travels a further 9 cm down, followed by 2 cm up and finally 6 cm down.

a What is the spider's position on the wall in relation to the light switch?
b What is the total distance travelled by the spider during this time?

Why did the teacher wear sunglasses?

Answer the questions below to find the puzzle code and my secret!

E = -6 + -8
=

E = +11 + -9
=

F = +2 + -11
=

H = +4 + -7
=

I = +2 + -5 + +2
=

L = -2 + -1 + -5
=

P = -8 + +18
=

R = -4 + +23
=

S = +5 + -17
=

T = -3 + -4 + -4
=

T = -5 + +12
=

U = -8 + +5 + -10
=

B = -2 + +5
=

E = +1 + +8
=

E = -7 + -13
=

F = -5 + +3
=

H = -1 + +7
=

J = -2 + +5 + +2
=

M = -5 + -10
=

Q = -9 + -7
=

R = -7 + +11
=

S = +13 + -20
=

T = -11 + +24
=

V = -24 + +5
=

C = +20 + -5
=

E = -12 + +7
=

F = -6 + +6
=

F = -25 + +8
=

I = -3 + -7
=

J = -15 + +27
=

O = +9 + -8
=

Q = +3 + -7
=

S = +1 + -4 + -3
=

T = -21 + +3
=

T = +20 + -4
=

X = -3 + -4 + +15
=

-18 +6 -14 -8 -20 +16 -11 -5 +19 +3 +2 -17 +1 +4 +9 -1 -2 -6

-16 -19 -4 +5 -15 +7 +8 0 -7 -9 +13 +10 +15 -12 +12 -3 -10 -13

Subtraction of integers

Consider the pattern:

$3 - 1 = 2$
$3 - 2 = 1$
$3 - 3 = 0$
$3 - 4 = -1$ and $3 + -4 = -1$

Subtracting a number gives the same result as *adding the opposite.*

$3 - 5 = -2$ and $3 + -5 = -2$

WORKED Example 8

Calculate the following:

a $2 - 5$ **b** $-3 - 6$
c $5 - -3$ **d** $-5 - -4$

THINK	WRITE
a 1 Write the question.	**a** $2 - 5$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= 2 - +5$ $= 2 + -5$
3 Add, using the addition rule for addition of integers.	$= -3$
b 1 Write the question.	**b** $-3 - 6$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= -3 - +6$ $= -3 + -6$
3 Add, using the addition rule for addition of integers.	$= -9$
c 1 Write the question.	**c** $5 - -3$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= 5 + +3$
3 Add, using the addition rule for addition of integers.	$= 5 + 3$ $= 8$
d 1 Write the question.	**d** $-5 - -4$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= -5 + +4$
3 Add, using the addition rule for addition of integers.	$= -5 + 4$ $= -1$

WORKED Example 9

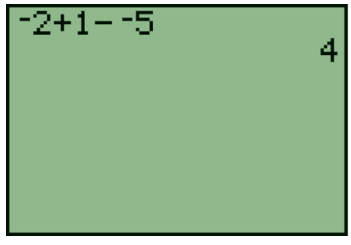
Evaluate the algebraic expression $a + b - c$, if $a = -2$, $b = 1$ and $c = -5$.

THINK	WRITE
1 Write the expression.	$a + b - c$
2 Replace each pronumeral with the appropriate integer.	$= -2 + 1 - -5$
3 Evaluate the expression.	$= -1 - -5$ $= -1 + +5$ $= 4$

Graphics Calculator tip! Entering a negative number

Entering a negative number

To enter a negative number into a graphics calculator, use the key marked (–). The keys to be entered for worked example 9 are: (–) 2 + 1 – (–) 5 EXE.



WORKED Example 10

A news flash in Freezonia announced that there had been a record drop in temperature overnight. At 6 pm the temperature was 10°C and by 4 am it had fallen 25°. What was the temperature at 4 am?

THINK	WRITE
1 Write the original temperature. Decide whether the temperature rose (addition) or fell (subtraction). Write the number sentence.	$10 - 25$ $= 10 + -25$
2 Find the answer.	$= -15$
3 Write the answer in a sentence.	The temperature in Freezonia at 4 am was –15°C.

remember

1. To subtract an integer, *add its opposite*: $a - b = a + -b$
2. Only the number *after* the subtraction symbol changes to its opposite.

$$3 - 5 = 3 + -5$$
$$= -2$$

EXERCISE 10E Subtraction of integers

Mathcad: Addition and subtraction of integers

1 Copy and complete the following.

a $3 - 8 = 3 + -8$
$= ____$

b $6 - 9 = 6 + - ____$
$= ____$

c $-2 - 5 = -2 + ____$
$= ____$

d $-9 - -8 = -9 + ____$
$= ____$

WORKED Example 8

2 Calculate the following.

a $7 - 5$ **b** $8 - -2$ **c** $-4 - 6$ **d** $-6 - -8$
e $1 - 10$ **f** $-5 - 5$ **g** $-8 - -8$ **h** $0 - 4$
i $0 - -3$ **j** $-10 - -20$ **k** $-11 - 3$ **l** $-5 - -5$

3 Are these number sentences true or false?

a $7 - 9 = 7 + -9$ **b** $0 - 8 = -8 - 0$ **c** $8 - 12 = -12 + 8$

EXCEL Spreadsheet: Subtraction of integers

4 Calculate the following mentally and write the answer.

a $-7 - 3$ **b** $8 - -5$ **c** $-6 - -9$ **d** $0 - -12$
e $-8 - 8$ **f** $3 - 20$ **g** $20 - -3$ **h** $-4 - 8$

5 Show working to calculate the following.
Remember: Order of operations: grouping symbols, × or ÷ moving from left to right, + or – moving from left to right.

a $6 + -3 - 2$ **b** $-9 + (5 - 7)$ **c** $3 - (8 - -2)$
d $6 + -8 + -5 - 2$ **e** $4 + (-8 + 10)$ **f** $16 - (-2 - -6)$

6 **multiple choice**

a $7 + -4 - -2$ is equal to:
A 9 **B** 1 **C** 13 **D** 5

b $6 - (2 + -3)$ is equal to:
A 1 **B** 7 **C** 11 **D** 5

7 Find the missing number in these incomplete number sentences.

a $8 + ____ = 0$ **b** $-2 + ____ = -8$ **c** $____ + -6 = 4$
d $____ + 5 = -2$ **e** $____ + -5 = -2$ **f** $____ - 7 = -6$
g $-8 - ____ = -17$ **h** $-8 - ____ = 17$ **i** $____ - -2 = 7$

WORKED Example 9

8 Evaluate each algebraic expression, if $a = 5$, $b = -2$, $c = -8$.

a $a + b$ **b** $b + c$ **c** $a - b$ **d** $b - c$ **e** $a + b + c$
f $a - b - c$ **g** $a - (b + c)$ **h** $c - b - a$ **i** $a - b + c$

WORKED Example 10

9 The temperature in the freezer was –20°C. Just before he went to bed, Dennis had a spoonful of ice-cream and left the freezer door ajar all night. The temperature in the front of the freezer rose 18° and the ice-cream melted. What was the temperature in the front of the freezer when his mother found the ice-cream in the morning?

10 Jill is climbing up a steep and slippery path to fetch a bucket of water. When she is 6 m above her starting point, she slips back 1 m, grasps some bushes by the side of the path and climbs 7 m more to a flat section. How far above her starting point is she when she reaches the resting place?

Multiplication of integers

Consider the patterns in the following multiplication tables.

$3 \times 3 = 9$
$3 \times 2 = 6$
$3 \times 1 = 3$
$3 \times 0 = 0$
$3 \times -1 = -3$
$3 \times -2 = -6$
$3 \times -3 = -9$

Result decreases by 3 each step ↓

positive × positive = positive
positive × negative = negative

$-3 \times 3 = -9$
$-3 \times 2 = -6$
$-3 \times 1 = -3$
$-3 \times 0 = 0$
$-3 \times -1 = 3$
$-3 \times -2 = 6$
$-3 \times -3 = 9$

Result increases by 3 each step ↓

negative × positive = negative
negative × negative = positive

When *multiplying* two integers with the *same sign* the answer is *positive*
When *multiplying* two integers with *different signs* the answer is *negative*.

Movies

The movie star can move either *east* (positive direction) or *west* (negative direction). The film can be run either *forwards* (positive) or *backwards* (negative). Fill in the chart showing the person's resulting direction of movement on the screen.

	Direction of the person's movement	Direction that the film is run	Resulting direction of the person's movement on the screen
1.	east (+)	forwards (+)	
2.	west (–)	forwards (+)	
3.	east (+)	backwards (–)	
4.	west (–)	backwards (–)	

Number pattern table

Copy and complete the table at right:

1. Fill in quadrant 2 first, by continuing the sequences 9, 6, 3, 0, and so on.
2. Then complete the shaded products involving 0.
3. Finally, complete quadrants 3 and 4, by continuing the sequences.

×	–3	–2	–1	0	1	2	3
3				0	3	6	9
2				0	2	4	6
1				0	1	2	3
0				0	0	0	0
–1							
–2							
–3							

WORKED Example 11

Evaluate: **a** $-5 \times +2$ **b** -4×-6 **c** $(-9)^2$ **d** $(-3)^3$ **e** $\sqrt[3]{-8}$

THINK

a 1 Write the question.
2 Negative × positive = negative.

b 1 Write the question.
2 Negative × negative = positive.

c 1 Write the question.
2 Write in extended notation.
3 Negative × negative = positive.

d 1 Write the question.
2 Write in extended notation.
3 Negative × negative = positive.
4 Positive × negative = negative.

e 1 Write the question.
2 Look for a number that when 3 lots of this number are multiplied together the result of –8 is obtained. The number must be negative for the cube of it to be negative.

WRITE

a $-5 \times +2$
$= -10$

b -4×-6
$= 24$

c $(-9)^2$
$= -9 \times -9$
$= 81$

d $(-3)^3$
$= -3 \times -3 \times -3$
$= +9 \times -3$
$= -27$

e $\sqrt[3]{-8}$
$-2 \times -2 \times -2 = -8$

So $\sqrt[3]{-8}$
$= -2$

remember

When multiplying integers:

Same direction gives a *positive* result

positive × positive = positive and negative × negative = positive

such as $+3 \times +2 = +6$ $-3 \times -2 = +6$.

Different directions give a *negative* result

positive × negative = negative and negative × positive = negative

such as $+3 \times -2 = -6$ $-3 \times +2 = -6$.

EXERCISE 10F Multiplication of integers

SkillSHEET 10.5

WORKED Example 11

1 Evaluate the following.

a 2×-5 **b** -6×3 **c** -7×9
d 6×-5 **e** -2×-3 **f** -4×-5
g -5×-5 **h** $(-6)^2$ **i** 0×-7
j -8×-1 **k** $(-7)^2$ **l** $\sqrt{16}$
m -24×1 **n** 10×-1 **o** -15×2
p -3×18 **q** -20×-10 **r** -6×-6
s $(-2)^3$ **t** $\sqrt[3]{-27}$ **u** $\sqrt[3]{-1}$

Mathcad Multiplication of integers

EXCEL Spreadsheet Multiplication of integers

2 Copy and complete the following.

a $-4 + -4 =$
$2 \times -4 =$
b $-2 + -2 + -2 =$
$3 \times -2 =$
c $-3 + -3 =$
$2 \times -3 =$
d $-5 + -5 + -5 + -5 =$
$4 \times -5 =$

3 Simplify the following.

a $2 \times -3 \times 4$ **b** $(-3)^2 \times 2$ **c** $3^2 \times -2$ **d** $(-2)^3$
e $-4 \times -3 \times 3$ **f** $-4 \times (-1)^3$ **g** $-8 \times 9 \times -2$ **h** $(-2)^4$

4 Formulate a rule for predicting the direction sign of any power with a negative base. Remember, in the number $(-2)^3$, -2 is the base and 3 is the power.

5 Simplify each algebraic expression.

a $3 \times 2 \times p$ **b** $-3 \times 4 \times t$ **c** $-2 \times -5 \times b$ **d** $2 \times a \times 4$
e $-3 \times c \times 5$ **f** $-2 \times d \times -7$ **g** $6 \times a \times -2 \times b$ **h** $-5x \times -2g$

6 Fill in the missing numbers.

a $-6 \times -3 =$ _____ **b** $6 \times$ _____ $= -18$ **c** _____ $\times 3 = -18$
d $-8 \times$ _____ $= -8$ **e** $-8 \times$ _____ $= 8$ **f** $-8 \times$ _____ $= 0$
g $-1 \times$ _____ $= 1$ **h** $(-$ _____$)^2 = 9$ **i** $-2 \times 3^2 =$ _____

7 **multiple choice**

Six people each owe the bank \$50. The combined total of the six accounts is:

A \$300
B –\$50
C \$50
D –\$300

GAME time Directed numbers and the number plane — 001

8 Dawn was taking her younger brother and sister to the local pool for a swim but she had spent all her money. It cost \$2 for each person so she borrowed the money from her parents. How much did she have if she swam too?

Division of integers

The division operation is the *inverse* or opposite of multiplication.

Since $3 \times 2 = 6$ then $6 \div 3 = 2$ and $6 \div 2 = 3$.
Since $3 \times -2 = -6$ then $-6 \div 3 = -2$ and $-6 \div -2 = 3$.
Since $-2 \times -3 = 6$ then $6 \div -3 = -2$ and $6 \div -2 = -3$.

Rules of division

1. *Same* directions gives a *positive* result:
   positive ÷ positive = positive and negative ÷ negative = positive
   such as $+6 \div +3 = +2$ and $-6 \div -3 = +2$.
2. *Different* directions give a *negative* result:
   negative ÷ positive = negative and positive ÷ negative = negative
   such as $-6 \div 3 = -2$ and $6 \div -3 = -2$.
3. Divide zero by any number and the answer is always zero.

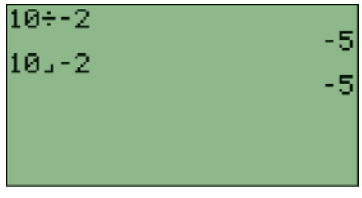
WORKED Example 12

Calculate: **a** $10 \div -2$ **b** $-12 \div 4$ **c** $-20 \div -5$.

THINK	WRITE
a 1 Write the question. 2 Look at the directions to decide whether the answer is positive or negative. If they are different, the answer is negative and if they are the same the answer is positive. Divide.	**a** $10 \div -2$ $= -5$
b 1 Write the question. 2 Different directions give a negative answer. Divide.	**b** $-12 \div 4$ $= -3$
c 1 Write the question. 2 Same directions give a positive answer. Divide.	**c** $-20 \div -5$ $= 4$

Graphics Calculator tip! Division with negative numbers

You can check your answers with a graphics calculator. Remember that negative numbers are entered using the (−) key. The ÷ key or fraction key $a^b/_c$ is used for operations on integers expressed as $10 \div -2$ as well as those expressed as $\frac{10}{-2}$.



WORKED Example 13

Simplify: **a** $\frac{-16}{+2}$ **b** $-5 \times \frac{4}{20}$.

THINK	WRITE
a 1 Write the question. ($\frac{-16}{+2}$ is the same as $-16 \div +2$)	**a** $\frac{-16}{+2}$
2 Divide using the type of directions to indicate the sign.	$= -8$
b 1 Write the question.	**b** $-5 \times \frac{4}{20}$
2 Write the integer as a fraction with a denominator of 1 and multiply numerators and denominators or cancel.	$= \frac{-5}{1} \times \frac{\cancel{4}^{1}}{\cancel{20}_{5}}$ $= \frac{-5}{5}$
3 Write the answer.	$= -1$

remember

Rules of division

1. The same directions or signs give a positive result.
2. Different directions or signs give a negative result.
3. Divide 0 by any integer and the answer is always zero.
4. $\frac{a}{b}$ is the same as $a \div b$.

EXERCISE 10G Division of integers

Mathcad: Division of integers

WORKED Example 12

1 Calculate the following.

a $-8 \div -2$ **b** $8 \div -2$ **c** $-8 \div 2$
d $12 \div -3$ **e** $-15 \div 5$ **f** $-16 \div -8$
g $-90 \div -10$ **h** $88 \div -11$ **i** $-6 \div 1$
j $-6 \div -1$ **k** $0 \div -4$ **l** $-84 \div 4$
m $-184 \div 2$ **n** $-125 \div -5$ **o** $-67 \div -1$
p $129 \div -3$ **q** $-284 \div 4$ **r** $336 \div -6$
s $0 \div -480$ **t** $-132 \div -11$ **u** $(-6)^2 \div -12$

EXCEL Spreadsheet: Division of integers

WORKED Example 13

2 Simplify the following.

a $\frac{-6}{2}$ **b** $\frac{-24}{-8}$ **c** $\frac{-8}{8}$ **d** $3 \times \frac{-2}{-6}$ **e** $4 \times \frac{-5}{10}$ **f** $-9 \times \frac{-3}{18}$

3 Fill in the missing numbers in these number sentences.

a $-21 \div$ _____ $= -7$ **b** _____ $\div -8 = -4$ **c** _____ $\div -9 = 8$
d $-11 \div$ _____ $= 1$ **e** _____ $\div -7 = 0$ **f** _____ $\div -4 = 4$
g $-42 \div$ _____ $= -6$ **h** $-96 \div$ _____ $= 2$ **i** $-150 \div -25 =$ _____

4 **multiple choice**

The missing numbers in the following number sentences could be:

a 16 ÷ _____ = _____

A 2, –8 **B** –2, –8 **C** –4, 4 **D** –2, 8

b _____ ÷ _____ = –5

A –15, 3 **B** 15, 5 **C** 25, 5 **D** –30, –6

5 A spider is running down the stairs from the first floor of an old lady's house to the basement below. It stops every 5 steps to catch a fly. If there are 26 steps above ground and 14 below, how many flies does the spider catch?

Combined operations

When simplifying numbers, the order of operations helps us to remember the correct order in which we should perform the various operations.

Order of operations
Working from left to right, calculate:
1. **within grouping symbols {[( )]} first**
2. **powers such as x^a next**
3. **then multiplication or division as it occurs, left to right × or ÷**
4. **and finally addition or subtraction as it occurs, left to right + or –.**

WORKED Example 14

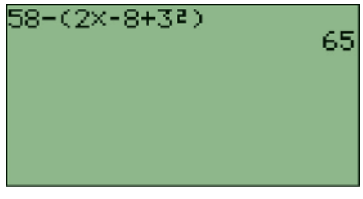
Calculate 58 – (2 × –8 + 3^2) using the correct order of operations.

THINK	WRITE
1 Write the question.	58 – (2 × –8 + 3^2)
2 Working inside grouping symbols, simplify the squared term.	= 58 – (2 × –8 + 9) *
3 Multiplication within the grouping symbols.	= 58 – (–16 + 9)
4 Addition within the grouping symbols.	= 58 – –7
5 When grouping symbols have been removed work the subtraction outside the grouping symbols.	= 58 + +7 *
	= 65

* These steps could be omitted with practice.

Graphics Calculator tip! Finding the square of a number

To calculate the square of a number such as 5^2, enter (5) then press the (x^2) and (EXE) keys. The screen opposite shows the calculation for worked example 14. Notice that you must enter grouping symbols so that the correct order of operations is applied.



Square of a number

remember

Order of operations

Work from left to right in the following order:
grouping symbols; powers; ×/÷; +/–

EXERCISE 10H Combined operations

SkillSHEET 10.6

WORKED Example 14

1 Calculate the following, using the correct order of operations.

a $6 + 3 \times -4$
b $18 - 12 \div -3$
c $8 + -4 - 10$
d $17 - 3 + -8$
e $6 \times -3 \div 9$
f $72 \div 8 \times -3$
g $7 + (-3 - 4)$
h $(6 + 3) \div -9$
i $-3 \times -2 + 3 \times -1$
j $-6 \times 5 - 2 \times -6$
k $-4 \times (6 - -4)$
l $(-8 + 3) \times -7$

Mathcad Combined operations

2 Evaluate each of the following.

a $4 + 7 \times -3 - 2$
b $-6 - 4 + (-3)^2$
c $(-2)^3 - 3 \times -2$
d $3 + (2 - 8) + -6$
e $-8 \div 2 + (-2)^2$
f $-4 \times -8 - [2 + (-3)^2]$
g $(-7 + 5) - -24 \div 6$
h $-15 \div (2 - 5) - 10$
i $54 \div -6 + 8 \times -9 \div -4$
j $(9 - -6) \div -5 + -8 \times 0$
k $-7 + -7 \div -7 \times -7 - -7$
l $-9 \times -5 - (3 - -2) + -48 \div 6$

3 **multiple choice**

The expression $6 + 2 \times -5 - -10 \div 2$ is equal to:

A –15 **B** –35 **C** –60 **D** 1

4 Model each situation with integers, then find the result:

a A submarine dives 100 m from sea level, rises 60 m then dives 25 m. What is its final position?

b Jemma has $274 in the bank, then she makes the following transactions: 2 withdrawals of $68 each, and then 3 deposits of $50 each.

c If 200 boxes of apples were each 3 short of the stated number of 40 apples, what was the overall shortfall in the number of apples?

d A person with a mass of 108 kg wants to reduce his mass to 84 kg in 3 months. What average mass reduction is needed per month?

e Local time in Sydney is 3 hours ahead of Singapore time, which is 5 hours behind Auckland (NZ) time. Auckland is 11 hours ahead of Berlin (Germany) time. What is the time difference between:

i Sydney and Berlin? **ii** Singapore and Berlin?

GAME time Directed numbers — 002

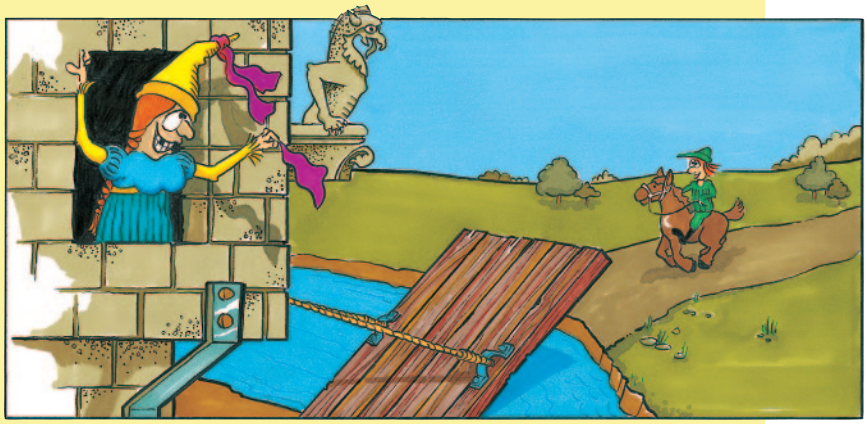
f Merlin is riding his bike east at a steady 10 km/h, while Morgan is riding her bike west at a steady 8 km/h. They pass each other on Backpedal bridge at 12 noon. (Assume that east is the positive direction and west is negative and that time *before* noon is negative and *after* noon is positive.)

i What is the location of each person with respect to the bridge at 9 am?
ii What are their locations with respect to the bridge at 2 pm?
iii How far apart were they at 10 am?
iv How far apart will they be at 4 pm?

WorkSHEET 10.2

Wet or dry reunion?

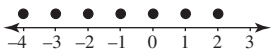
Marion looks out of the window of her castle and spies Robin, 100 m down the road, riding towards her at 2 m/s. She immediately starts to lower the drawbridge.



1. If the drawbridge is 10 m high and takes 60 s to lower, what is the average change, each second, in the height of the drawbridge?
2. If Robin does not slow down as he approaches the castle, will he make it into the castle? Give reasons.
3. If the point where the edge of the moat meets the road is taken as zero, and Robin is +100 m from the moat when Marion first sees him, how far is he from the edge of the moat when the drawbridge hits the ground?
4. Robin's horse is 2.8 m tall. If the distance from the horse's head to the surface of the water when the horse stands in the moat is 0.7 m, how far is the bottom of the moat from the surface of the water?

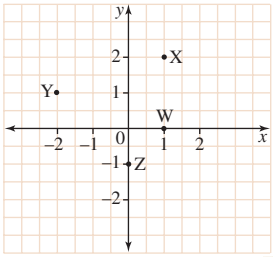
10 QUICK QUESTIONS 2

1. Describe the integers graphed on the number line to the right.



2. On a number line, graph the integers > –3.

Use the number plane at right for questions **3** and **4**.



3. Give the coordinates and quadrant for the point Y.
4. Name the point fitting the description (1, 0).
5. Arrange these numbers in descending order: 0, 12, –4, 5, –11.
6. List the next 3 integers in the sequence: –54, –45, –36, _____, _____, _____.
7. Describe a situation to fit the number sentence $4 + -5 = -1$.
8. Find the missing numbers: $-7 -$ _____ $-$ _____ $= -11$.
9. Evaluate $2 \times (-3)^2 \times -4$.
10. Evaluate $55 \div -5 + 7 \times -6 \div -2$.

Directed numbers on the number line

Positive and negative numbers have both size and direction, so they are often called *directed numbers*. Directed numbers are all values on the number line. They include zero, positive and negative whole numbers and fractions (common and decimal).

WORKED Example 15

Graph each of the following sets of directed numbers on a number line.

a $x > -2\frac{1}{2}$ **b** $x \leq 1\frac{3}{4}$ **c** $-2\frac{1}{3} < x \leq 3.4$

THINK

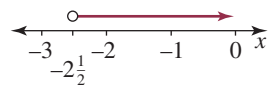
a > means 'is greater than', so x takes on all values to the right of $-2\frac{1}{2}$. Use an open or unshaded circle to indicate that the number itself is not included.

b ≤ means 'is less than or equal to', so x takes on all values to the left of and including $1\frac{3}{4}$. Show all points fitting the description. Use a closed or shaded circle to show that the number itself is included.

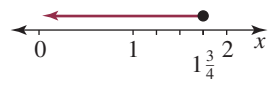
c $-2\frac{1}{3} < x \leq 3.4$ means 'all numbers lying between the given boundaries, and including 3.4 itself'. Use an open dot to show that $-2\frac{1}{3}$ is not included and a closed dot to show that 3.4 is included. Use estimation to locate the boundary points.

WRITE

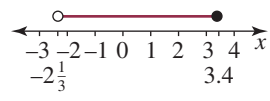
a



b



c



remember

1. **Symbols for directed numbers on a number line**

   _____ > _____ means 'is greater than' or 'is to the right of'

   _____ < _____ means 'is less than' or 'is to the left of'

   _____ ≤ _____ means 'is less than or equal to' or 'is to the left of and including'

   _____ ≥ _____ means 'is greater than or equal to' or 'is to the right of and including'

   _____ < _____ < _____ means 'is between but not including'.
2. When graphing number lines an open or unshaded circle indicates that the number is not included. A closed or shaded circle indicates that the number is included.

EXERCISE 10I Directed numbers on the number line

1 State true or false:

a Integers are directed numbers.
b Fractions are directed numbers.
c Decimals are directed numbers.
d -3.6 is the opposite of 3.6.
e $-\frac{3}{4}$ is a negative fraction.
f Zero is neither negative nor positive.
g $\frac{1}{2} < -\frac{3}{4}$
h $-8.2 > 2.1$
i $0 > -6\frac{1}{3}$
j $-7\frac{1}{2} < -7\frac{1}{4}$

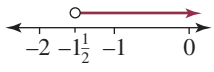
WORKED Example 15

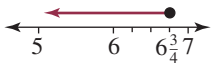
2 Graph each of the following sets of directed numbers on a number line.

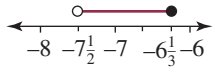
a $x > -3\frac{1}{2}$
b $x < -2\frac{1}{3}$
c $x \geq -4.5$
d $x \leq 6.25$
e $-2 < x < 1$
f $-1\frac{2}{3} < x \leq \frac{1}{2}$
g $-6\frac{3}{4} \leq x < -4\frac{1}{2}$
h $x \geq 2.75$
i $x < -1\frac{4}{5}$
j $2\frac{3}{5} \leq x \leq 3.8$

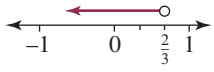
Mathcad Directed numbers on the number line

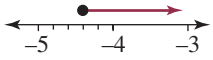
3 Describe the directed numbers graphed on each number line. Use number sentences such as: $x > -2\frac{1}{2}$, $x \leq 4.8$, and so on.

a


b


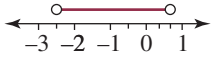
c


d


e


f

g

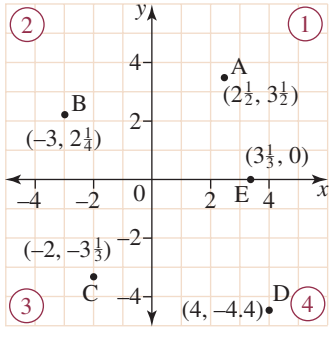
h


4 Arrange the following in ascending order.

a $6, -3, 0, 5.8, -4\frac{1}{2}$
b $3\frac{2}{3}, -1, -4.2, 1\frac{1}{2}, -\frac{3}{4}$
c $0, -\frac{2}{3}, \frac{1}{4}, -6.8, -8.6$

Directed numbers on the number plane

The four quadrants of the number plane divided by the horizontal (x-) axis and the vertical (y-) axis can display points with ordered pairs made up of any directed numbers.



A in the first quadrant is the point $(2\frac{1}{2}, 3\frac{1}{2})$.

B in the second quadrant is the point $(-3, 2\frac{1}{4})$.

C in the third quadrant is the point $(-2, -3\frac{1}{3})$.

D in the fourth quadrant is the point $(4, -4.4)$.

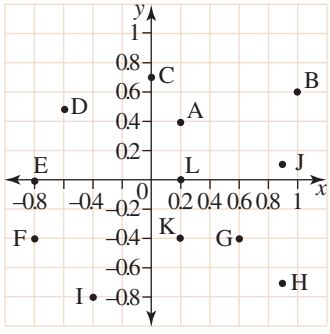
E on the x-axis is the point $(3\frac{1}{3}, 0)$.

The entire surface of the number plane can now be referred to, rather than just the integer lattice or grid.

EXERCISE 10J Directed numbers on the number plane

Mathcad Directed numbers on the number plane

Questions **1**–**10** relate to this diagram.



1 Write the coordinates and state the quadrant or axis location of each point.
a A **b** B **c** C **d** G **e** I

2 Name the point on the number plane above and give its quadrant/axis.
a (–0.8, –0.4) **b** (–0.6, 0.5) **c** (0.9, 0.1)
d (–0.8, 0) **e** (0.9, –0.7) **f** (0.2, –0.4)

3 Give the x-coordinate of the points at:
a A **b** K **c** L.

4 Give the y-coordinate of:
a F **b** K **c** G.

5 What can you say about points that have the same x-coordinate?

6 What can you say about points that have the same y-coordinate?

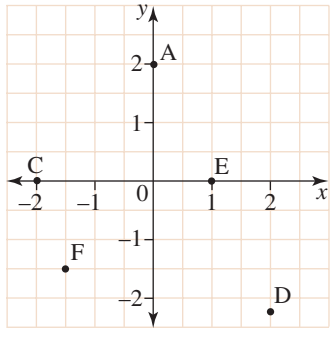
7 What is the x-coordinate of all points on the y-axis?

8 What is the y-coordinate of all points on the x-axis?

9 Name all points in the fourth quadrant.

10 True or false?
a $(0, 1\frac{2}{3})$ must lie on the y-axis. **b** (0, 0) is the origin.
c (0, –2.8) must lie on the x-axis. **d** (–0.2, 0.8) is the same point as (0.8, –0.2).
e (–0.6, 0.4) is in the third quadrant. **f** J and H have the same y-coordinate.

11 **a** Using the diagram at right, find the coordinates of a point, B, such that ABCO is a square.
b Find the coordinates of a point, G, such that ACOG is a parallelogram.
c Give the coordinates of F.
d Estimate the coordinates of D.
e Give the coordinates of a point 1 unit away from A.



Directed number operations: fractions

The rules for addition, subtraction, multiplication and division of integers apply to all directed number operations and are illustrated by the following examples.

For whole numbers

Addition

To add integers:

1. of the *same* signs, add and keep the sign: $-2 + -4 = -6$
2. of *different* signs, find the difference between the signless numbers, then use the sign of the number further from zero: $-8 + 2 = -6$
3. of *opposite* signs, result is zero: $-4 + 4 = 0$

Subtraction

To subtract integers: add the opposite: $6 - -8 = 6 + 8 = 14$

Multiplication and division

To multiply or divide integers:

1. of the *same* signs, give a positive result: $-3 \times -2 = 6$, $-8 \div -4 = 2$
2. of *different* signs, give a negative result: $-5 \times 2 = -10$, $-10 \div 2 = -5$

For fractions

Addition and subtraction

Write all fractions with the same denominator, then add or subtract numerators:

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Multiplication

Cancel the common factors in numerators and denominators, then multiply the numerators and multiply the denominators.

$$\frac{\cancel{3}^{1}}{\cancel{8}^{2}} \times \frac{\cancel{4}^{1}}{\cancel{9}^{3}} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

Division

Change ÷ to × then tip the divisor (times and tip), then follow the multiplication rules.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{1} = \frac{3 \times 1}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$$

WORKED Example 16

Calculate $-\frac{1}{3} + \frac{1}{2}$.

THINK	WRITE
1 Write the expression.	$-\frac{1}{3} + \frac{1}{2}$
2 Write both fractions with the same denominator.	$= -\frac{2}{6} + \frac{3}{6}$
3 Write as one fraction.	$= \frac{-2+3}{6}$
4 Add the numerators. They are of different signs, so find the difference and use the sign of the number further from zero.	$= \frac{1}{6}$

WORKED Example 17

Calculate $-\frac{3}{4} - \frac{1}{6}$.

THINK	WRITE
1 Write the expression.	$-\frac{3}{4} - \frac{1}{6}$
2 To subtract, add the opposite or inverse.	$= -\frac{3}{4} + -\frac{1}{6}$
3 Write both fractions with the same denominator.	$= -\frac{9}{12} + -\frac{2}{12}$
4 Add the numerators. (They are the same sign so add and keep the sign.)	$= -\frac{11}{12}$

WORKED Example 18

Simplify $\frac{2}{5} \times -\frac{5}{8}$.

THINK	WRITE
1 Write the expression and cancel the common factors in numerators and denominators.	$\frac{\cancel{2}^{1}}{\cancel{5}^{1}} \times \frac{-\cancel{5}^{-1}}{\cancel{8}^{4}}$
2 Multiply numerators and multiply denominators.	$= \frac{1 \times -1}{1 \times 4}$
	$= -\frac{1}{4}$

WORKED Example 19

Evaluate $-\frac{3}{4} \div -1\frac{1}{2}$.

THINK	WRITE
1 Write the expression using a division sign.	$-\frac{3}{4} \div -1\frac{1}{2}$
2 Change the divisor to an improper fraction.	$= -\frac{3}{4} \div -\frac{3}{2}$
3 Change ÷ to ×, tip the divisor (times and tip) and cancel common factors in numerators and denominators.	$= -\frac{\cancel{3}^{1}}{\cancel{4}^{2}} \times -\frac{\cancel{2}^{1}}{\cancel{3}^{1}}$
4 Multiply numerators and denominators.	$= -\frac{1}{2} \times -\frac{1}{1}$
	$= \frac{1}{2}$

remember

Addition

Write all fractions with the same denominator, then add numerators. If the sign is the same, add and keep the sign. If the sign is different, then find the difference between the signless numbers and use the sign furthest from zero.
Opposites add to zero.

Subtraction

Write all fractions with the same denominator, then subtract numerators by adding the opposite.

Multiplication

Cancel common factors in numerators and denominators then multiply numerators and multiply denominators. If the signs are the same, the result is positive. If the signs are different, the result is negative.

Division

Change ÷ to × and invert the divisor, then follow the rules for multiplication.
If mixed numbers are involved in multiplication and division of fractions, change them to improper fractions.

EXERCISE 10K Directed number operations: fractions

WORKED Example 16

1 Calculate.

a $-\frac{3}{5} + \frac{1}{5}$

b $-\frac{3}{8} + -\frac{5}{8}$

c $\frac{1}{4} + -\frac{1}{2}$

d $-\frac{1}{6} + -\frac{2}{3}$

e $\frac{1}{2} + -\frac{3}{8}$

f $-\frac{1}{2} + -\frac{1}{3}$

g $-\frac{3}{5} + \frac{1}{2}$

h $\frac{3}{4} + -\frac{1}{3}$

i $-2\frac{1}{2} + -1\frac{3}{4}$

j $1\frac{2}{3} + -2\frac{3}{5}$

k $-2\frac{1}{2} - 3\frac{1}{3}$

l $3\frac{1}{4} + 1\frac{3}{5}$

WORKED Example 17

2 Calculate.

a $\frac{1}{2} - \frac{3}{4}$

b $-\frac{1}{2} - \frac{1}{3}$

c $-\frac{1}{3} - \frac{2}{5}$

d $-\frac{3}{4} - -\frac{2}{5}$

e $2\frac{1}{2} - 3\frac{1}{4}$

f $-3\frac{3}{5} - -1\frac{1}{3}$

SkillSHEET 10.7

EXCEL Spreadsheet: Four operations with fractions

Mathcad: Directed number operations: fractions

WORKED Example 18

3 Simplify each of the following.

a $-\frac{1}{2} \times \frac{1}{3}$

b $-\frac{3}{4} \times -\frac{1}{5}$

c $-\frac{2}{3} \times 7$

d $\frac{1}{3} \times -\frac{3}{4}$

e $-\frac{3}{4} \times \frac{5}{6}$

f $-\frac{5}{6} \times \frac{3}{10}$

g $-\frac{8}{9} \times 1\frac{3}{4}$

h $-3\frac{1}{7} \times -\frac{7}{8}$

i $-2\frac{1}{2} \times \frac{4}{5}$

WORKED Example 19

4 Evaluate each of thefollowing.

a $-\frac{1}{5} \div \frac{1}{2}$

b $\frac{2}{3} \div -\frac{3}{4}$

c $-\frac{7}{4} \div -\frac{2}{1}$

d $\frac{3}{2} \div -4$

e $-\frac{1}{8} \div \frac{3}{4}$

f $-1\frac{4}{5} \div 6$

g $-2\frac{1}{4} \div -\frac{1}{2}$

h $2\frac{2}{3} \div -1\frac{1}{9}$

i $-\frac{3}{5} \div 2\frac{5}{8}$

5 Evaluate each of the following.

a $-\frac{2}{3} + \frac{1}{6} \times -\frac{2}{5}$

b $1\frac{1}{2} \times -\frac{5}{6} \div \frac{4}{7}$

c $-\frac{7}{8} \div -1\frac{3}{4} - \frac{1}{2}$

d $\left(\frac{2}{5} - \frac{6}{7}\right) \times -3\frac{1}{3}$

e $\left(-1\frac{1}{2} - 3\frac{4}{5}\right) \div \frac{3}{5}$

f $\frac{9}{10} \times -\frac{5}{3} \div \left(1\frac{2}{7} - 2\frac{1}{2}\right)$

Directed number operations: decimals

The rules for operations with decimals are illustrated by the following examples.

Positive decimals

Addition

$$\begin{array}{r} 3.28 \\ +1.50 \\ \hline 4.78 \end{array}$$

Subtraction

$$\begin{array}{r} {}^{5}\cancel{6}.{}^{1}8 \\ -0.\ 9 \\ \hline 5.\ 9 \end{array}$$

Check the answer by rounding: $3.28 + 1.5 \approx 3 + 2 = 5$ and $6.8 - 0.9 \approx 7 - 1 = 6$.

Multiplication 6.4×0.03

$$\begin{array}{rl} 64 & \text{1 decimal place} \\ \times\ 3 & \text{2 decimal places} \\ \hline 192 & (1 + 2) = 3 \text{ decimal places} \end{array}$$

$6.4 \times 0.03 = 0.192$

Division

Change the divisor to a whole number.

$$\begin{aligned} 0.018 \div 0.04 &= (0.018 \times 100) \div (0.04 \times 100) \\ &= 1.8 \div 4 \\ &= 0.45 \end{aligned}$$

Apply the directed number rules to decimals.

WORKED Example 20

Calculate: a –3.64 + –2.9 b –5.7 + 2.4 and check each answer by using estimation.

THINK	WRITE
a 1 They have the same sign, so add and keep the sign.	**a** $\begin{array}{r} -3.64 \\ +\ -2.90 \\ \hline -6.54 \\ \hline \end{array}$
2 Check the answer by using estimation.	$-4 + -3 = -7$
b 1 Different signs, so subtract the smaller number from the larger number and use the sign of the number further from zero.	**b** $\begin{array}{r} 5.7 \\ -\ 2.4 \\ \hline 3.3 \end{array}$ $-5.7 + 2.4 = -3.3$
2 Check the answer by using estimation.	$-6 + 2 = -4$

WORKED Example 21

Calculate –5.307 – 0.62 and check the answer by using estimation.

THINK	WRITE
1 Write the question.	$-5.307 - 0.62$
2 Change to addition of the opposite.	$= -5.307 + -0.62$
3 Calculate.	$\begin{array}{r} -5.307 \\ +\ -0.620 \\ \hline -5.927 \\ \hline \end{array}$
4 Check the answer by using estimation.	$-5 - 1 = -6$

WORKED Example 22

Simplify –3.8 × 0.05.

THINK	WRITE
Multiply as for whole numbers.	$\begin{array}{r} 3^{4}8 \\ \times\ 5 \\ \hline 190 \\ \hline \end{array}$
Count the number of decimal places in the question and insert the decimal point. The signs are different so insert a negative sign in the answer.	$-3.8 \times 0.05 = -0.190$ $= -0.19$

WORKED Example 23

Find the quotient of –0.015 ÷ –0.4 giving an exact answer.

THINK	WRITE
1 Write the question.	$-0.015 \div -0.4$
2 Multiply both parts by 10 to produce a whole number divisor.	$= -0.15 \div -4$
3 Divide until an exact answer is achieved or until a recurring pattern is evident.	$\begin{array}{r} 0.03\ 7\ 5 \\ 4\,\overline{)\,0.15^{3}0^{2}0} \end{array}$
4 The signs are the same so the answer is positive.	$-0.015 \div -0.4 = 0.0375$

EXERCISE 10L Directed number operations: decimals

SKILLSHEET 10.8

WORKED Example 20

1 Calculate the following and check each answer by using estimation.

a $-0.4 + -0.5$
b $0.2 + -0.9$
c $-0.8 + 0.23$
d $-0.021 + -0.97$
e $13.69 + -6.084$
f $-0.0037 + 0.638$

Mathcad Directed number operations: decimals

WORKED Example 21

2 Calculate the following and check each answer by using estimation.

a $0.8 - 1.5$
b $-0.6 - 0.72$
c $-3 - -6.4$
d $-2.6 - 1.7$
e $-3.2 - -0.65$
f $0.084 - 0.902$

WORKED Example 22

3 Simplify each of the following.

a 0.3×-0.2
b -0.8×0.9
c -0.4×-0.06
d $(-0.6)^2$
e $(-0.3)^2$
f -0.004×40
g 4000×-0.5
h -0.02×-0.4
i $(-0.05)^2$
j -4.9×0.06
k $(0.2)^2 \times -40$
l $(1.2)^2 \times (0.3)^2$

WORKED Example 23

4 Find the quotient of each of the following, giving an exact answer.

a $-8.4 \div 0.2$
b $0.15 \div -0.5$
c $-0.0405 \div -0.3$
d $-15 \div 0.5$
e $0.049 \div -0.07$
f $-3.2 \div -0.008$
g $-0.0036 \div 0.06$
h $270 \div -0.03$
i $-0.04 \div -800$
j $0.8 \div -0.16$
k $(1.2)^2 \div 0.04$
l $(1.5)^2 \div 0.05$

WORKSHEET 10.3

MATHS QUEST CHALLENGE

1 **Give an example of two numbers that fit each of the descriptions which follow. If no numbers fit the description, explain why.**
   a **Both the sum and the product of two numbers are negative.**
   b **The sum of two numbers is positive and the quotient is negative.**
   c **The sum of two numbers is 0 and the product is positive.**

2 **On a test, each correct answer scores 5 points, each incorrect answer scores –2 points, and each question left unanswered scores 0 points.**
   a **Suppose a student answers 16 questions correctly, 3 questions incorrectly and does not answer 1 question. Write an expression for the student's score and find the score.**
   b **Suppose you answer all 20 questions on the test. What is the greatest number of questions you can answer incorrectly and still get a positive score? Explain your reasoning.**

Mountain climbing

Remember the description of the mountain climbers from page 351? From their base camp, the group travelled 506.3 m upwards. One climber started to suffer altitude sickness and was escorted down by another climber to a point 273.1 m below the base camp.

1 Show the relative positions of the two groups on a vertical number line.
2 How far apart were the two groups?
3 How far would the smaller group need to travel to catch up with the main group if the main group has climbed a further 89.8 m?
4 If the smaller group takes 5.5 hours to reach the base camp, how many hours of climbing time will be needed to reach the main group who are waiting for them? Assume they are climbing at a constant rate.

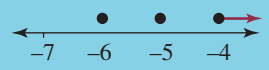
summary

Copy the sentences below. Fill in the gaps by choosing the correct word or expression from the word list that follows.

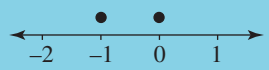
1 **Integers are _____ and negative whole numbers, together with zero. They have both size and _____.**

2 **Negative two (–2) is the _____ of positive two (+2). Values increase as we move to the _____ of the number line. $-6 < -2$ and $1 > -8$**

3 **The number line at right shows the integers which are _____ than –7.**

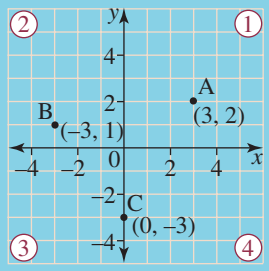


4 **The number line at right shows integers _____ –2 and 1.**



5 **Ascending order means from smallest to largest; the reverse is _____ order.**

6 **This integer number lattice has a _____ axis called the x-axis and a _____ axis called the y-axis. The point (0, 0) is called the _____. Four quarters or _____ make up the grid. Each point is located by two _____ in the order (x, y).**



7 **Directed numbers include the _____ together with positive and negative common fractions and _____.**

8 **Rules of operation are:**

			Example
Add	+	**the *same* sign, keep the sign and add:**	$-7 + -3 = -10$
		_____ signs, find the difference and use the sign of the number further from zero:	$-7 + 3 = -4$
Subtract	–	**add the opposite:**	$3 - 7 = 3 + -7 = -4$
Multiply	×	**the _____ sign gives a positive result:**	$-3 \times -2 = 6$
		***different* signs give a negative result:**	$-3 \times 2 = -6$
Divide	÷	**as for multiplication:**	$6 \div -3 = -2$; $-6 \div -3 = 2$

WORD LIST

right	direction	greater	coordinates
descending	quadrants	integers	opposite
decimal numbers	horizontal	same	origin
vertical	different	positive	between

CHAPTER review

1 Which of the following are integers? **10A**

a $-2\frac{1}{2}$ **b** 0.45 **c** 0 **d** −201

2 Complete each statement by inserting the correct symbol >, < or =. **10A**

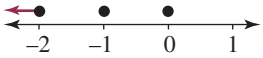
a −6 −2 **b** −7 7 **c** 0 −5 **d** −100 9

3 List the integers between −21 and −15. **10A**

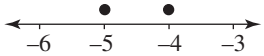
4 Arrange in *descending* order: −3, 2, 0, −15. **10A**

5 Describe the integers graphed on each number line: **10A**

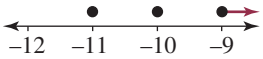
a



b



c



6 Graph each set of integers on a number line: **10A**

a integers between −7 and −2

b integers > −3 **c** integers ≤ −4.

7 State whether the following points are on the *x*-axis, the *y*-axis, both axes or in the first quadrant. **10B**

a (0, 0) **b** (0, 5) **c** (3, 0) **d** (3, 2)

8 Draw and label appropriately a set of axes. Plot the following points in the order given, joining each point to the next one. Name the shape that has been drawn. **10C**

(−2, 3), (1, 3), (2, −2), (−1, −2), (−2, 3)

9 In which quadrant or on which axes do the following points lie? **10C**

a (−2, 3) **b** (3, −1) **c** (−4, −1) **d** (0, 2) **e** (−1, 0) **f** (7, 9)

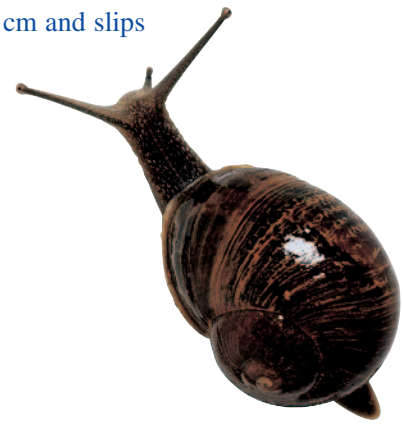
10 Calculate the following. **10D**

a −12 + 7 **b** −9 + −8 **c** 18 + −10 **d** 5 + 1

11 Write the number that is 2 more than each of the following integers. **10D**

a −4 **b** 5 **c** −1 **d** 0 **e** −2

12 A snail begins to climb up the side of a bucket. It climbs 3 cm and slips back 2 cm, then climbs a further 4 cm and slips back 1 cm. Write a number sentence to help you find how far the snail is from the bottom of the bucket. **10D**



13 Calculate the following. **10E**

a −5 − 3 **b** 17 − −9 **c** −6 − −9 **d** 6 − 8

10F **14** Evaluate the following.

a -6×7 **b** 4×-8 **c** -2×-5 **d** $(-8)^2$

10G **15** Calculate the following.

a $-36 \div 3$ **b** $-21 \div -7$ **c** $45 \div -9$ **d** $\frac{-18}{-2}$

10H **16** Calculate the following.

a $10 - 6 \times 2$ **b** $-7 - -8 \div 2$ **c** $-3 \times -5 - -6 \times 2$
d $(-2)^3$ **e** $(-3 - 12) \div (-10 + 7)$

10H **17** Model this situation with integers, then find the result: A scuba diver at 52 metres below sea level made his ascent in 3 stages of 15 metres each. At what level was he then?

10I **18** Write the next 3 numbers of the sequence.
$-3.6, -2.4, -1.2,$ _____, _____, _____

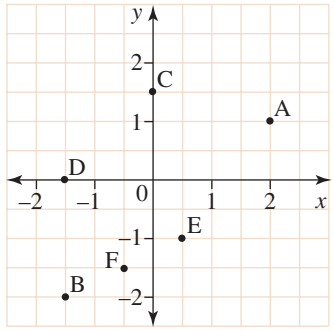
10I **19** Describe the directed numbers graphed on each number line.

a

b

10I **20** Graph the directed numbers $x > -2\frac{3}{4}$ on a number line.

Questions **21** to **24** relate to the figure below right.



10J **21** Give the coordinates and quadrant or axis for each point.

a A **b** B **c** C

10J **22** Name the point for each set of coordinate pairs.

a $(\frac{1}{2}, -1)$ **b** $(-1\frac{1}{2}, 0)$ **c** $(-0.5, -1.5)$

10J **23** Give the coordinates of the midpoint of interval BD.

10J **24** Give the coordinates of a point, G, which would form a parallelogram with B, D and E.

10K **25** Calculate.

a $-1\frac{19}{60} + \frac{1}{4}$ **b** $-\frac{3}{5} - \frac{7}{10}$ **c** $-\frac{7}{8} \times \frac{5}{14}$ **d** $-2\frac{3}{4} \div -\frac{3}{8}$

10L **26** Calculate, giving an exact answer.

a $-2.48 + 1.903$ **b** $-1.63 - 2.54$ **c** 0.08×-0.4 **d** $-1.02 \div -0.5$

test yourself CHAPTER 10